Dyson-Schwinger Equations and Quantum Chromodynamics

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http://www.phy.anl.gov/theory/staff/cdr.html

Room with a View

Universal Truths
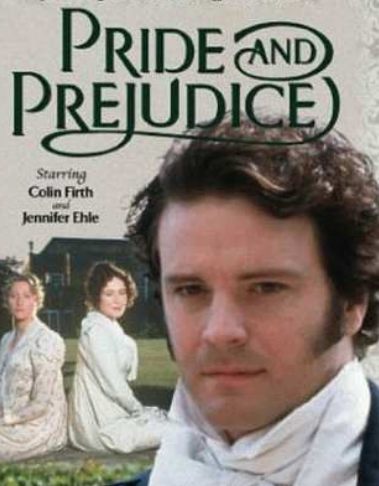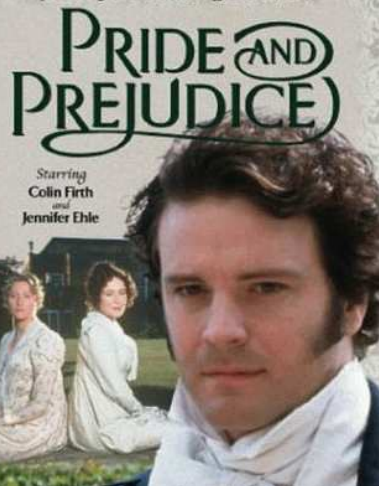
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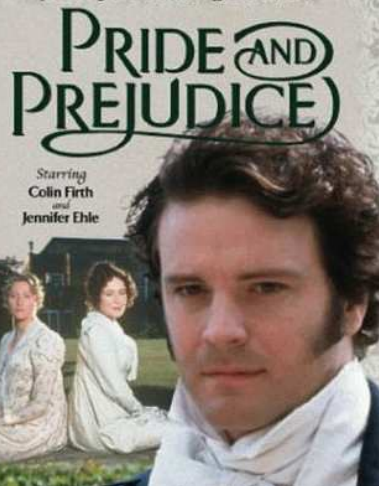
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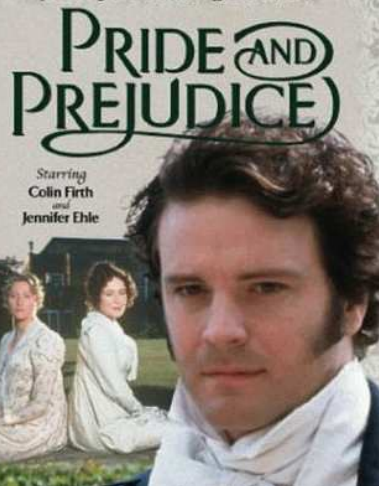
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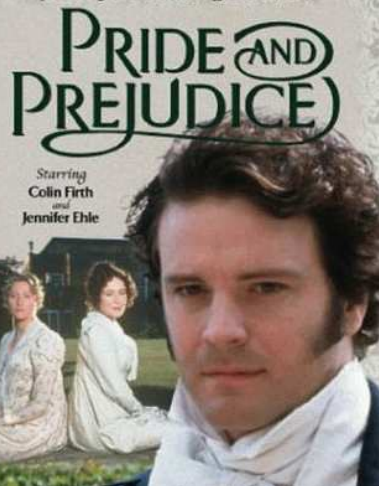
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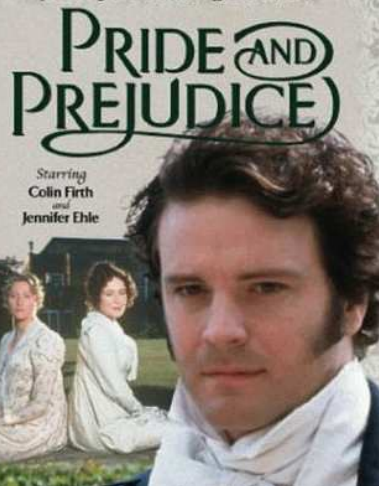
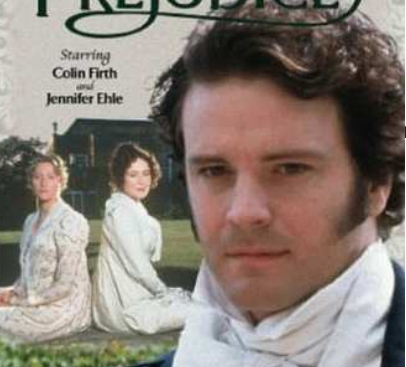- DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.

Form Factors: Why?

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- Experimental and theoretical studies of nucleon electromagnetic form factors have made rapid and significant progress during the last several years, including new data in the time like region, and material gains have been made in studying the pion form factor.

- Despite this, many urgent questions remain unanswered.

- What is the role of pion cloud in nucleon electromagnetic structure?

- Can we understand the pion cloud in a more quantitative and, perhaps, model-independent way?

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Where is the transition from non-pQCD to pQCD in the pion and nucleon electromagnetic form factors?

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- Do we understand the high Q^2 behavior of the proton form factor ratio in the space-like region?

- Can we make model-independent statements about the role of relativity or orbital angular momentum in the nucleon?

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- Can we understand the rich structure of the time-like proton form factors in terms of resonances?

- What do we expect for the proton form factor ratio in the time-like region?

- What is the relation between proton and neutron form factor in the time-like region?

- How do we understand the ratio between time-like and space-like form factors?

- What is the role of two-photon exchange contributions in understanding the discrepancy between the polarization and Rosenbluth measurements of the proton form factor ratio?

- What is the impact of these contributions on other form factor measurements?

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How accurately can the pion form factor be extracted from the $ep \rightarrow e'n\pi^+$ reaction?

Status

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- Current status is described in

  - J. Arrington, C. D. Roberts and J. M. Zanotti
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    J. Phys. G **34**, S23 (2007); [arXiv:nucl-th/0611050].

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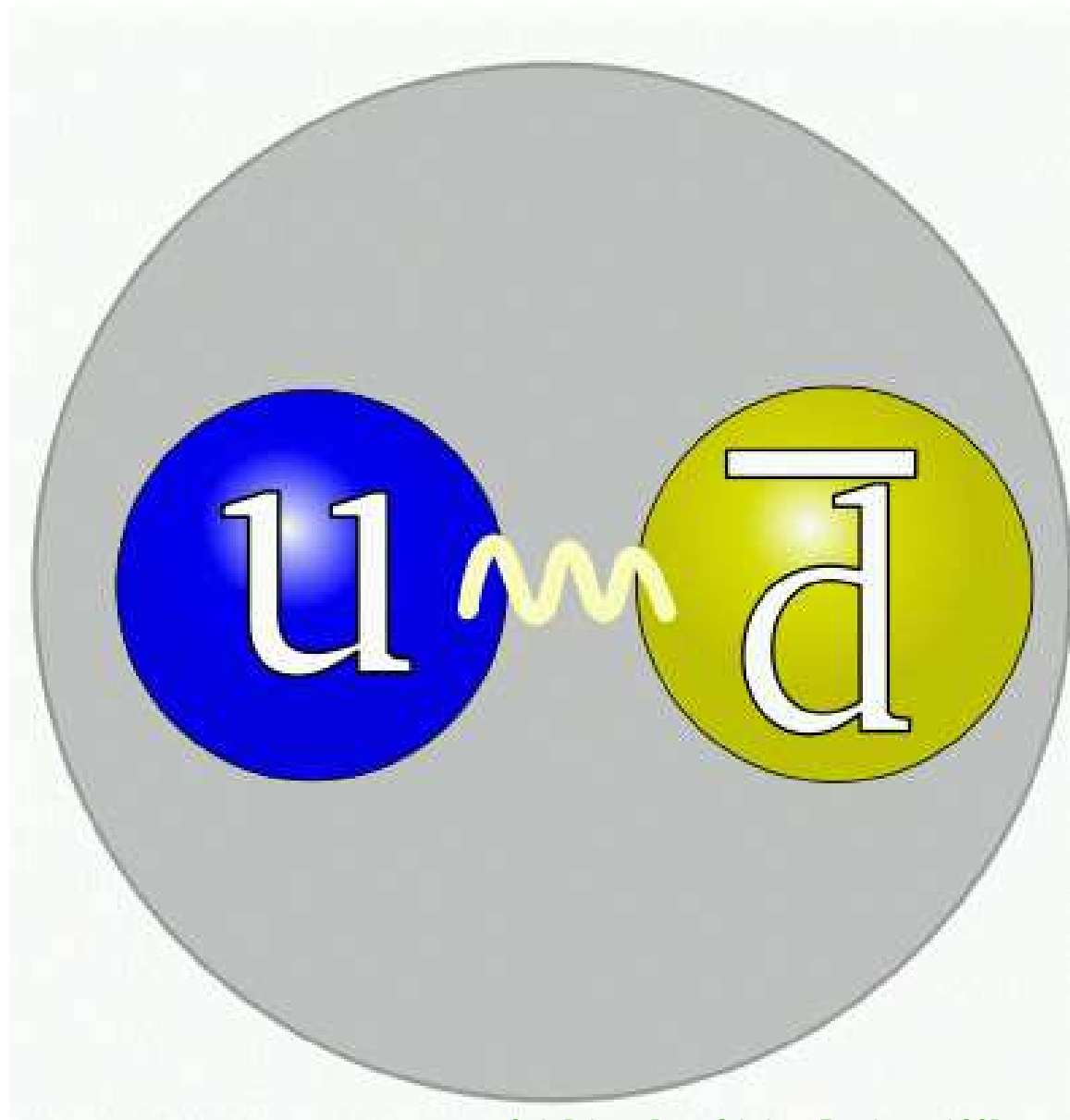
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- Most recently:
  "ECT* Workshop on Hadron Electromagnetic Form Factors"
  Organisers: Alexandrou, Arrington, Friedrich, Maas, Roberts
  Presentations, etc., available on-line
  http://ect08.phy.anl.gov/
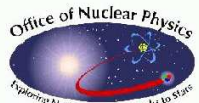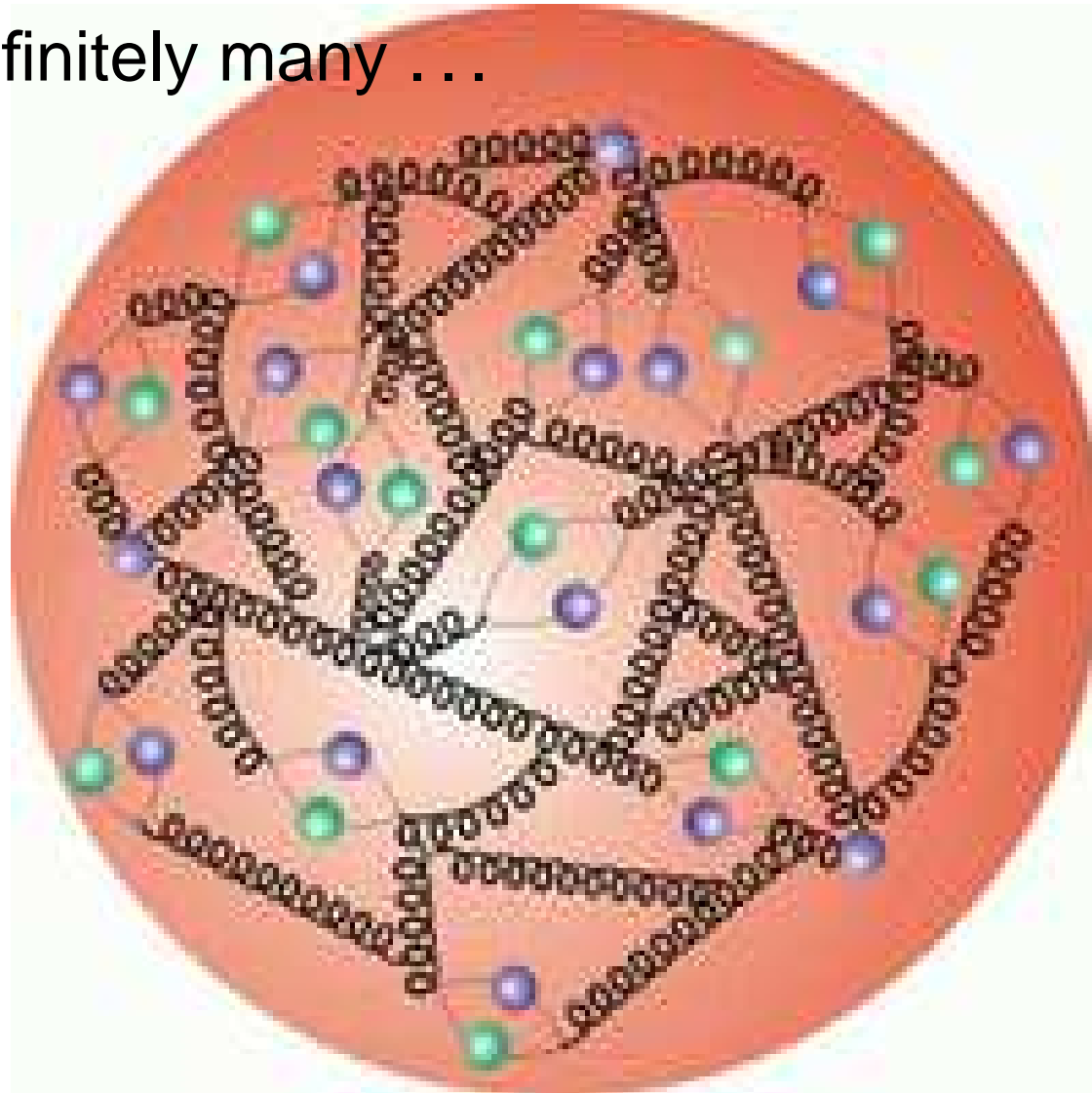
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Two → Infinitely many . . .

Two → Infinitely many …
Handle that
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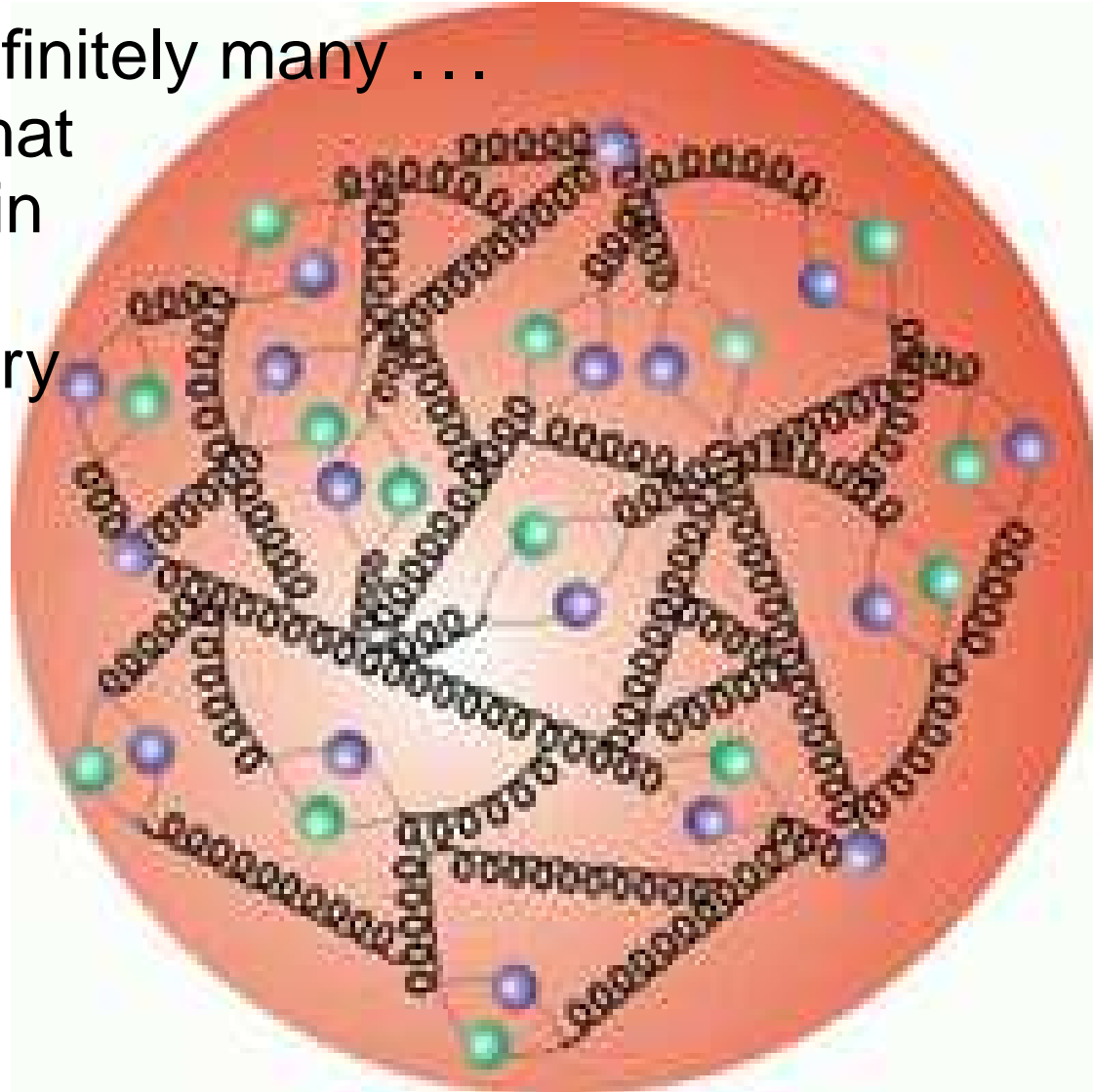
Two → Infinitely many …
Handle that properly in quantum field theory … momentum -dependent dressing
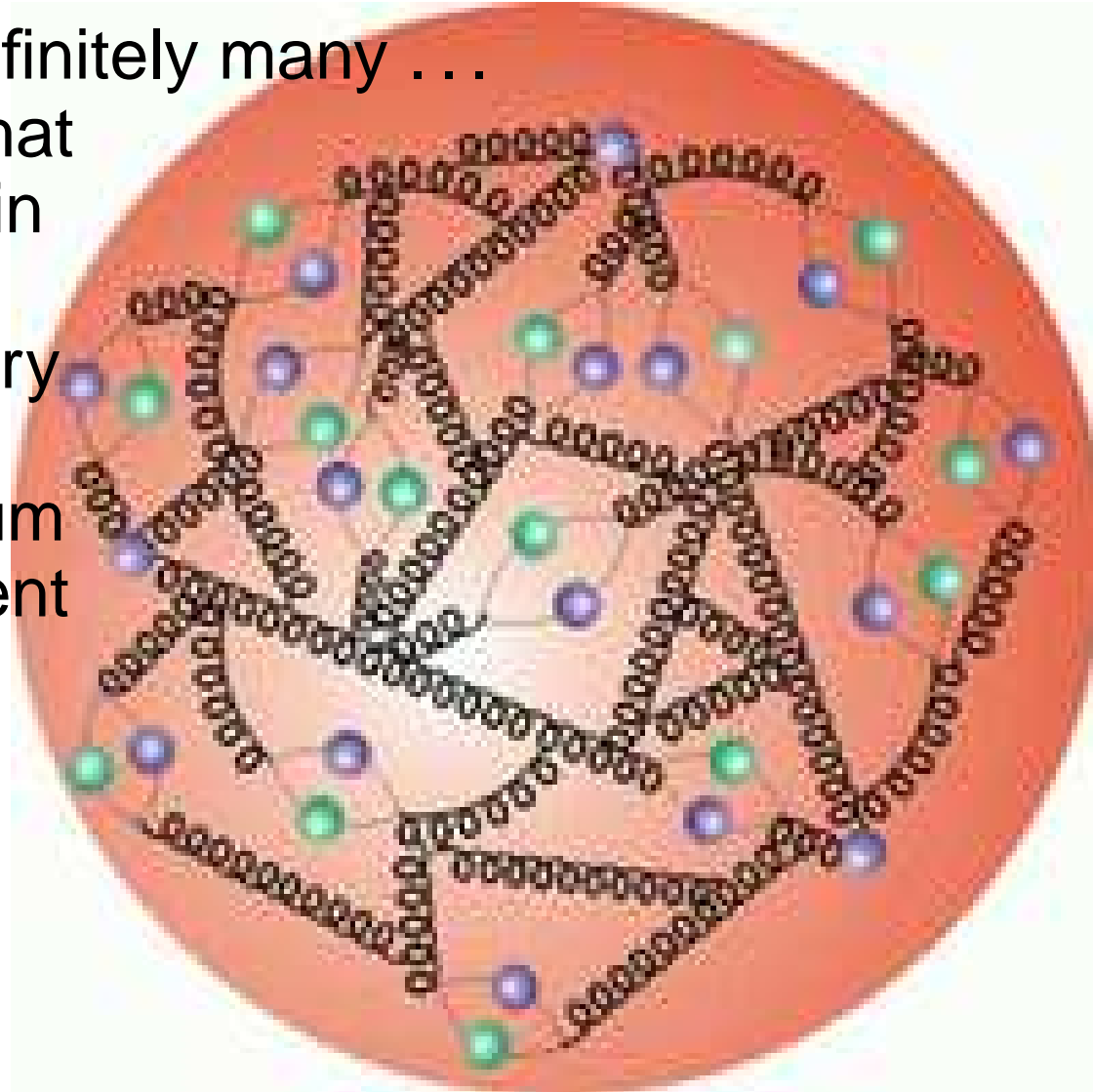
Two → Infinitely many . . .
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properly in
quantum
field theory
. . .
momentum
-dependent
dressing
. . .
perceived
distribution of
mass depends
on the resolving scale
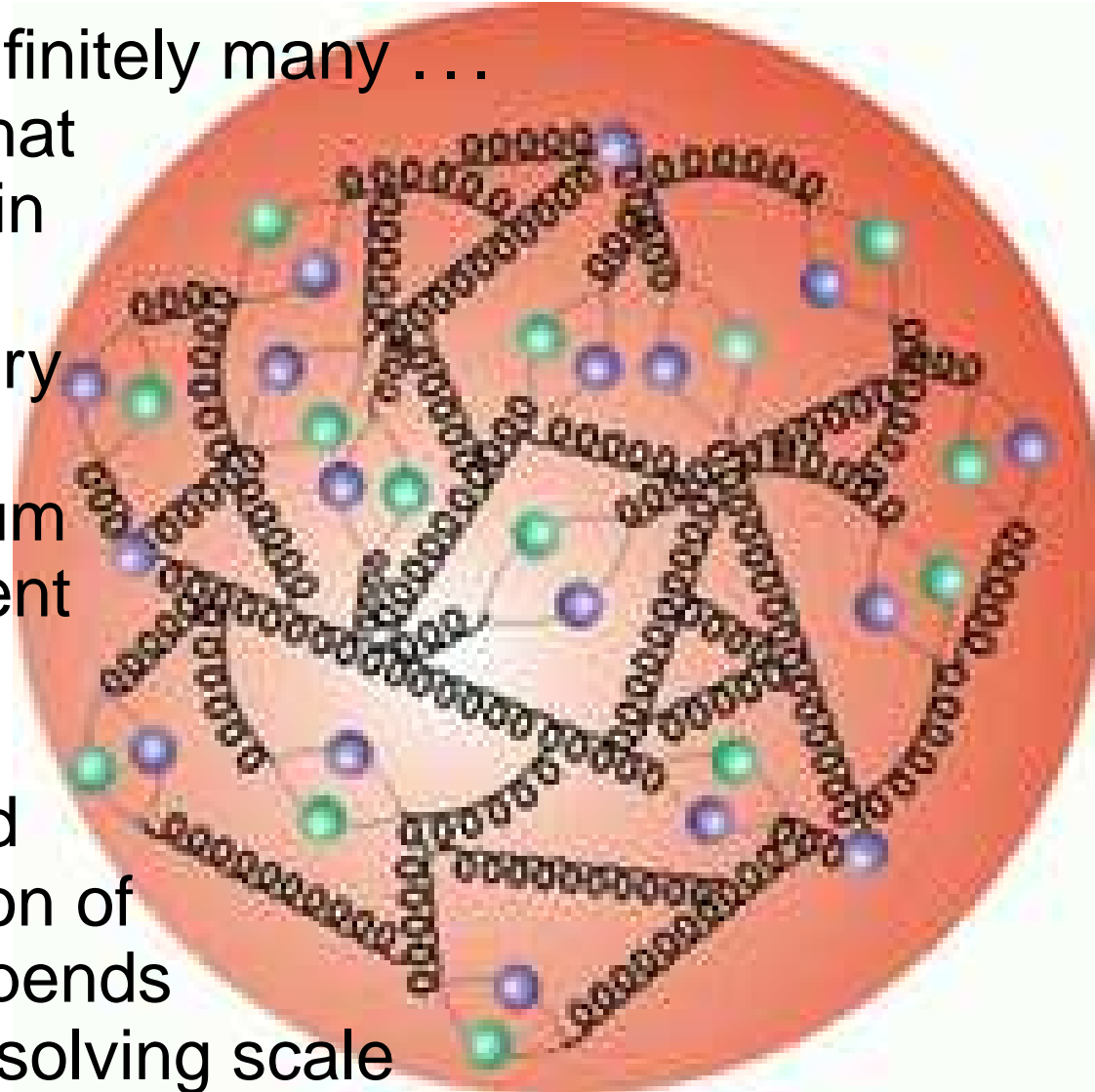
Pion Form Factor

Procedure Now Straightforward

- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$

- Use that to Complete Bethe Salpeter Kernel, K

- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π
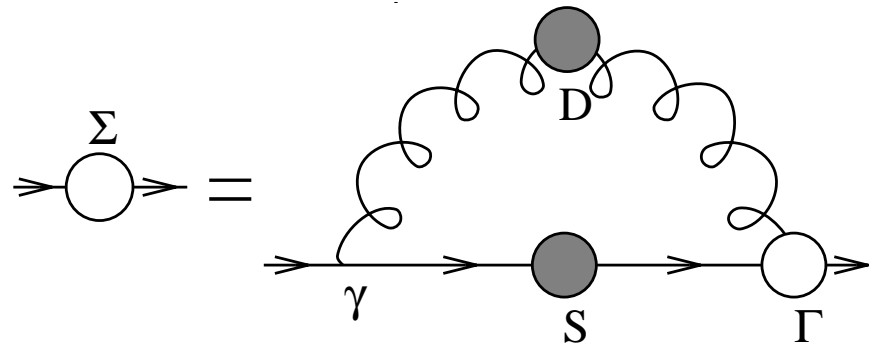
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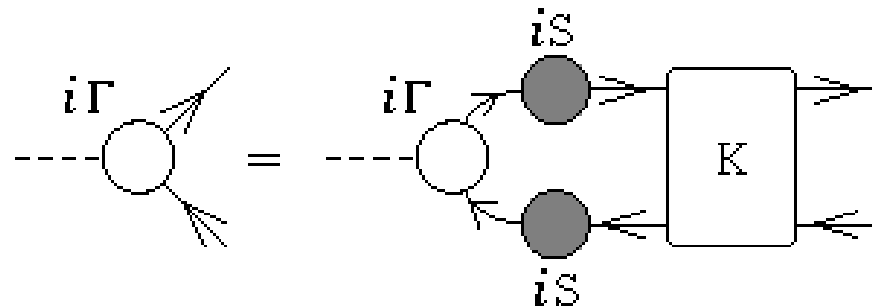
- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex, Γ_μ

Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor

$\Gamma_\pi(k; P)$

$\Gamma_\mu(k; P)$

$S(p)$

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$\Gamma_\pi(k; P)$

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$S(p)$
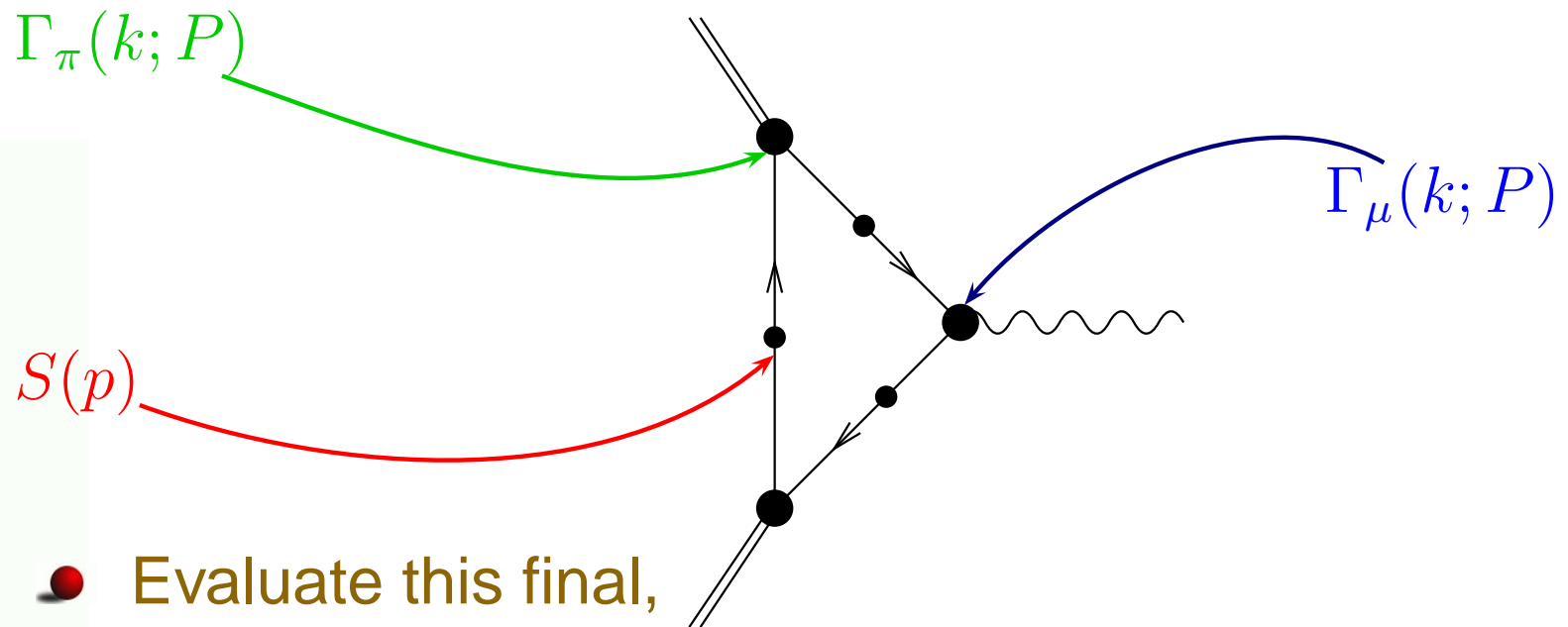
- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied

Numerical method improved in 2005
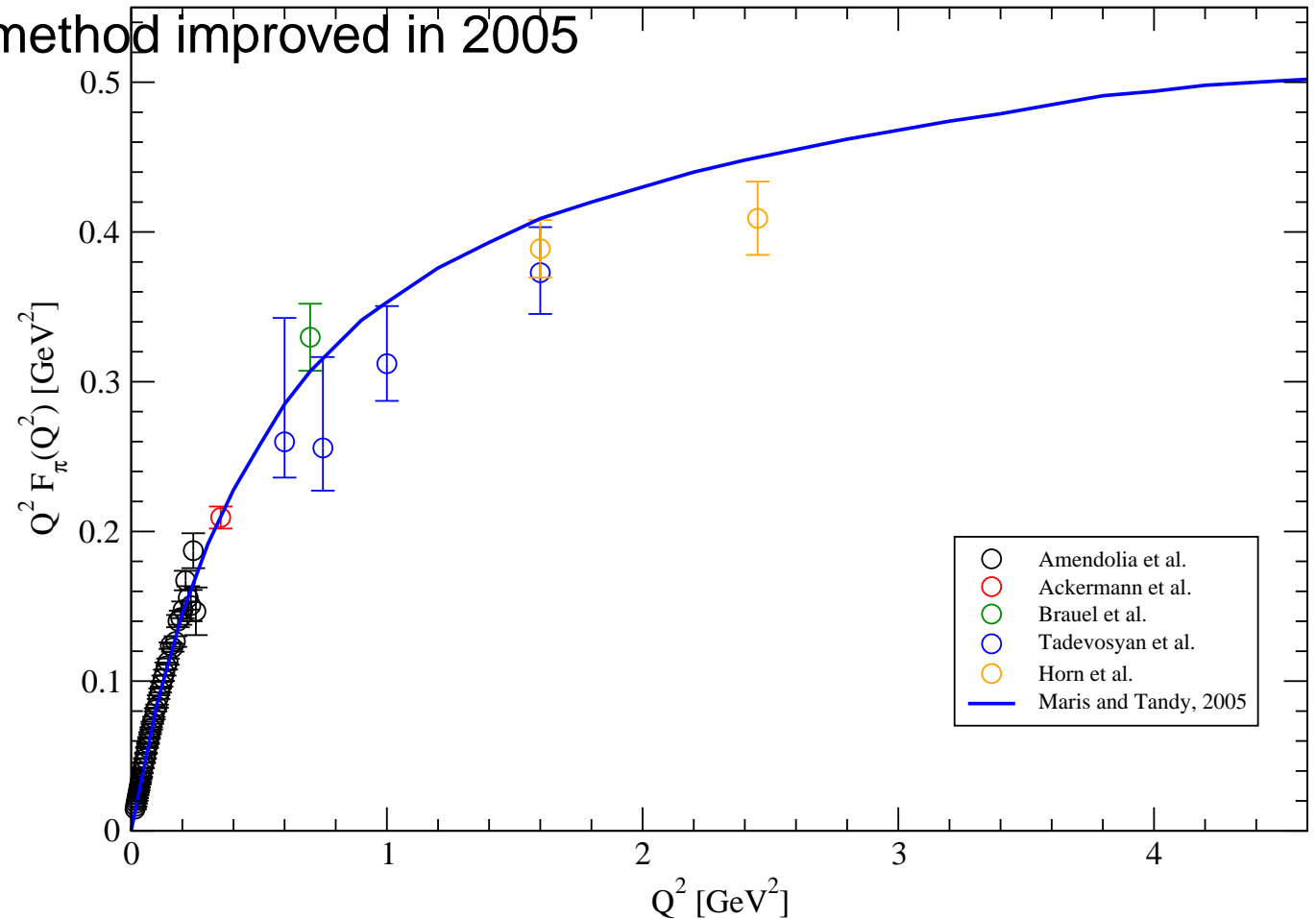
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Data published
in 2001.
Subsequently
revised

Legend:
- ○ Amendolia et al.
- ○ Ackermann et al.
- ○ Brauel et al.
- ○ Tadevosyan et al.
- ○ Horn et al.
- —— Maris and Tandy, 2005

x-axis: Q^2 [GeV^2]
y-axis: $Q^2 F_\pi(Q^2)$ [GeV^2]
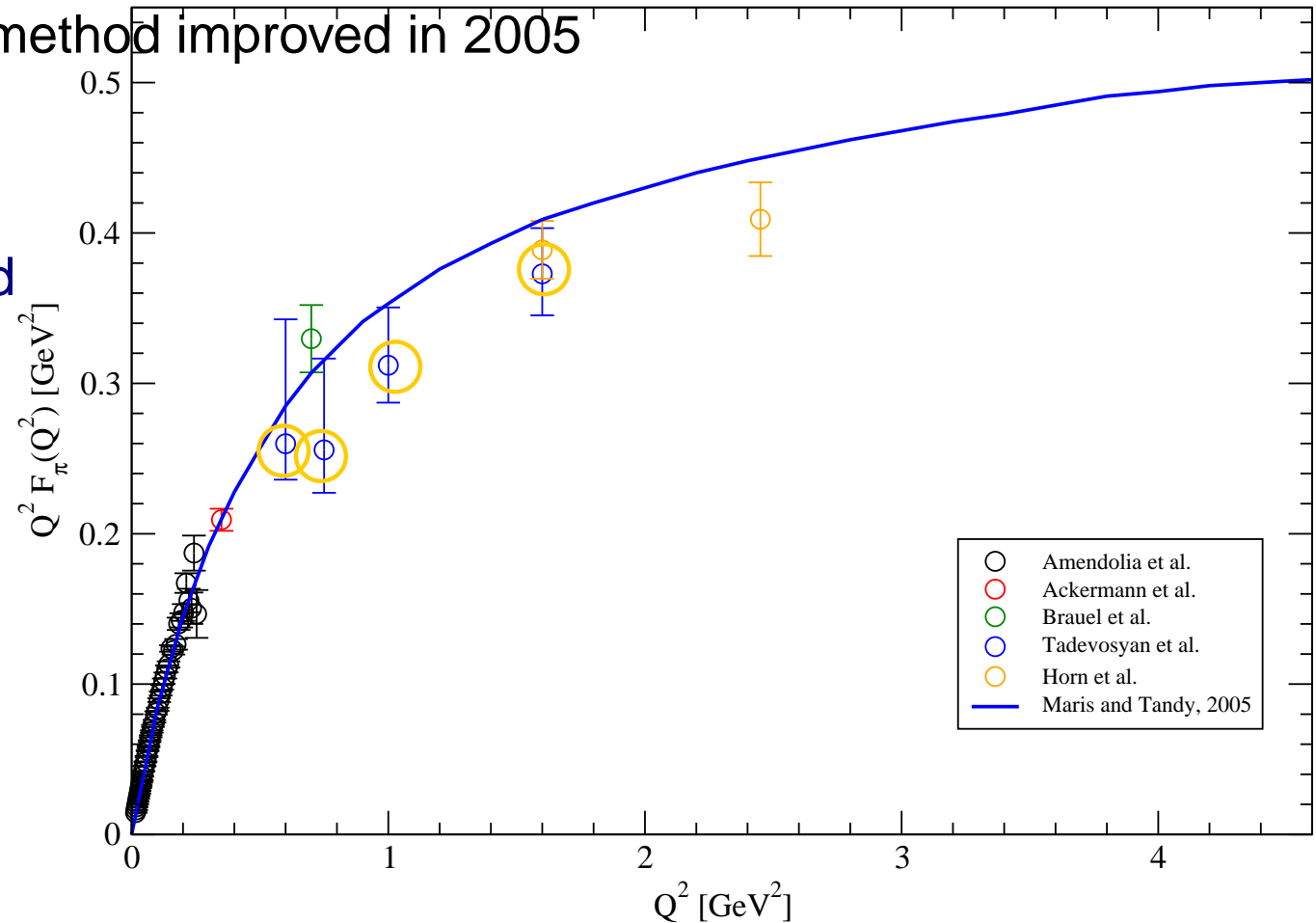
Timelike Pion Form Factor

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Timelike Pion Form Factor

Ab initio calculation into timelike region

Deeper than ground-state ρ-meson pole

Timelike **Pion Form Factor**

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Deeper than ground-state ρ-meson pole
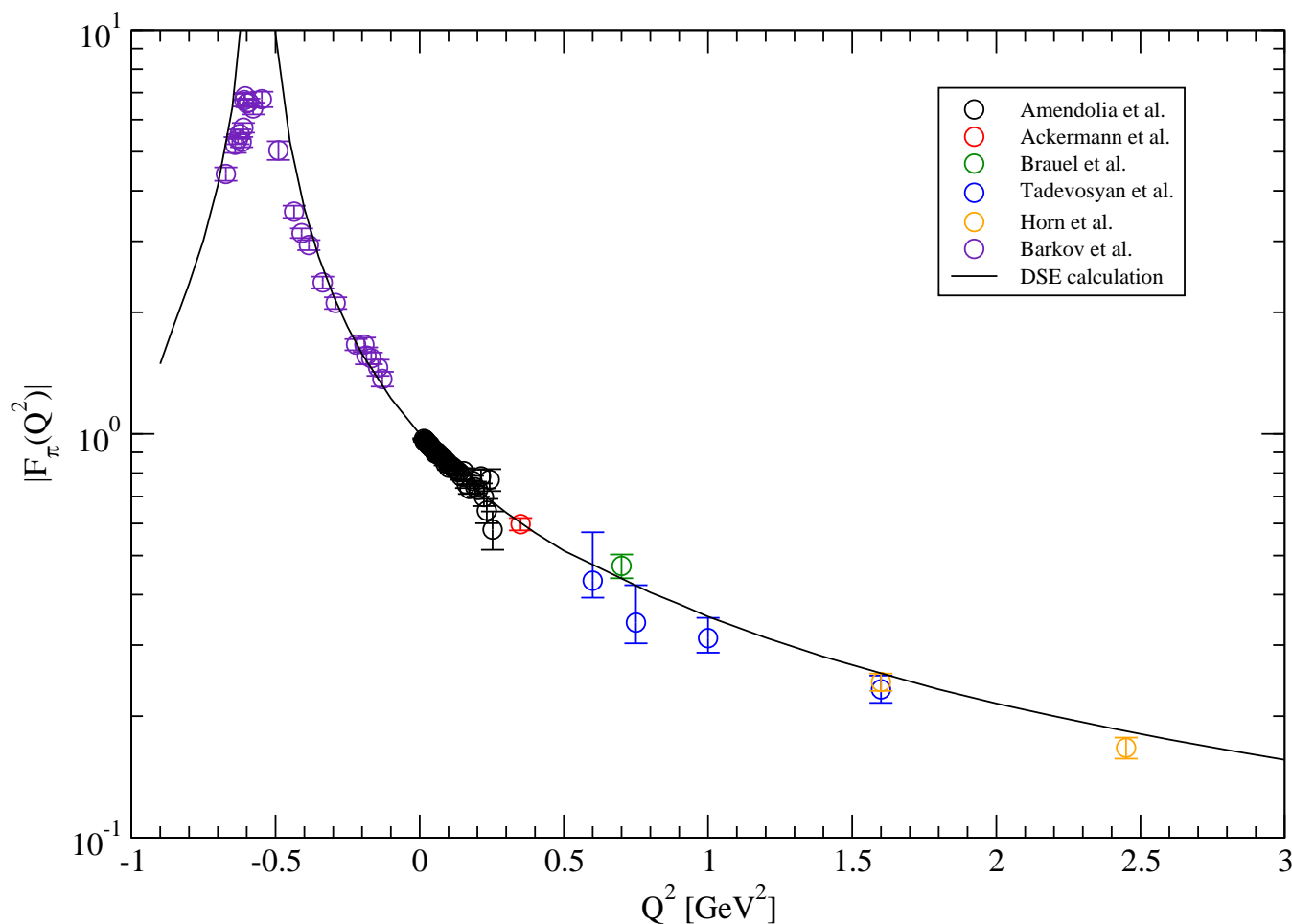
Timelike Pion Form Factor

Ab initio calculation into timelike region

Deeper than ground-state ρ-meson pole

ρ-meson not put in "by hand" − generated dynamically as a bound-
state of dressed-quark and dressed-antiquark
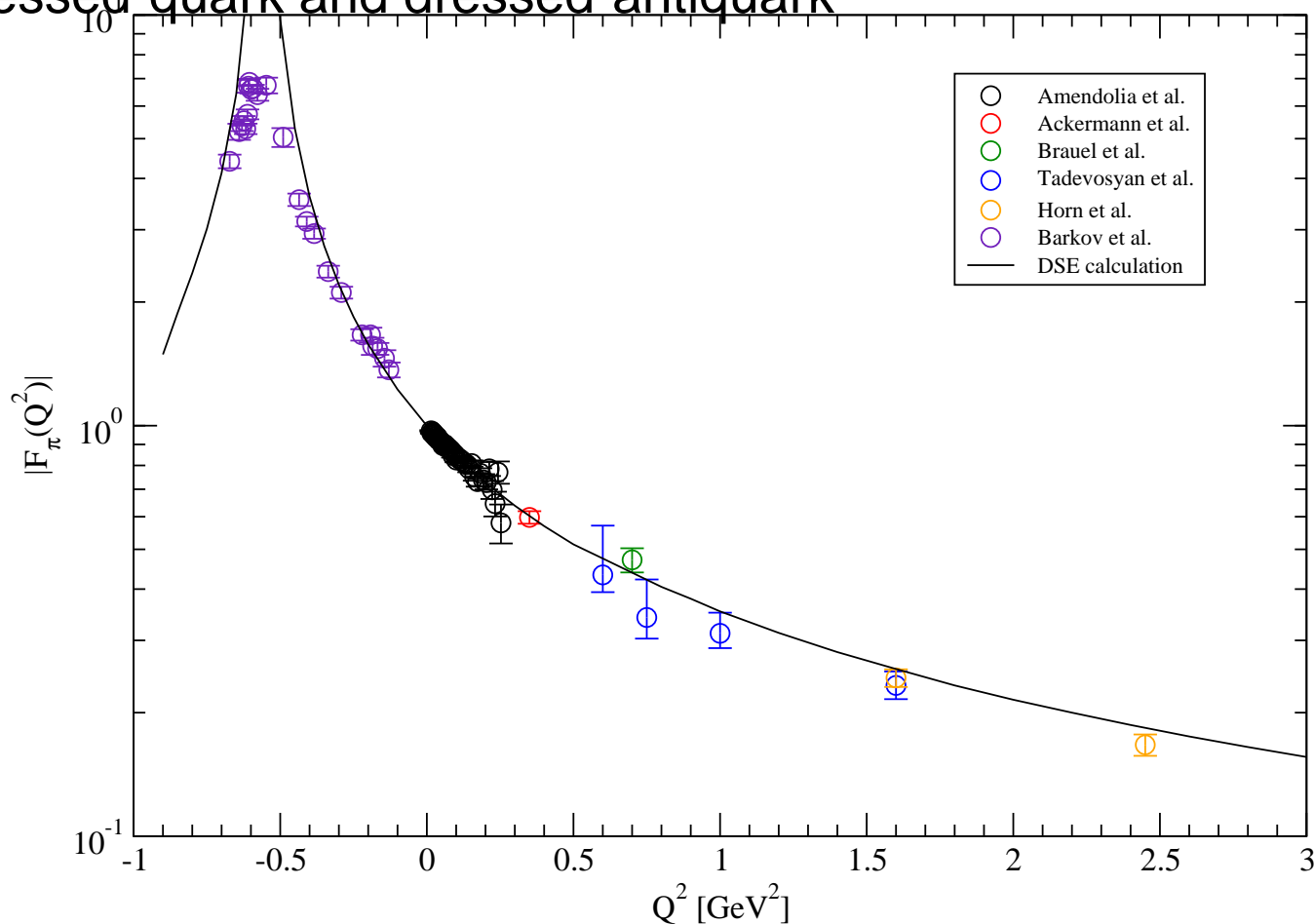
Dimensionless product: $r_\pi \, f_\pi$

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- Great strides towards placing nucleon studies on same footing as mesons

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- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$

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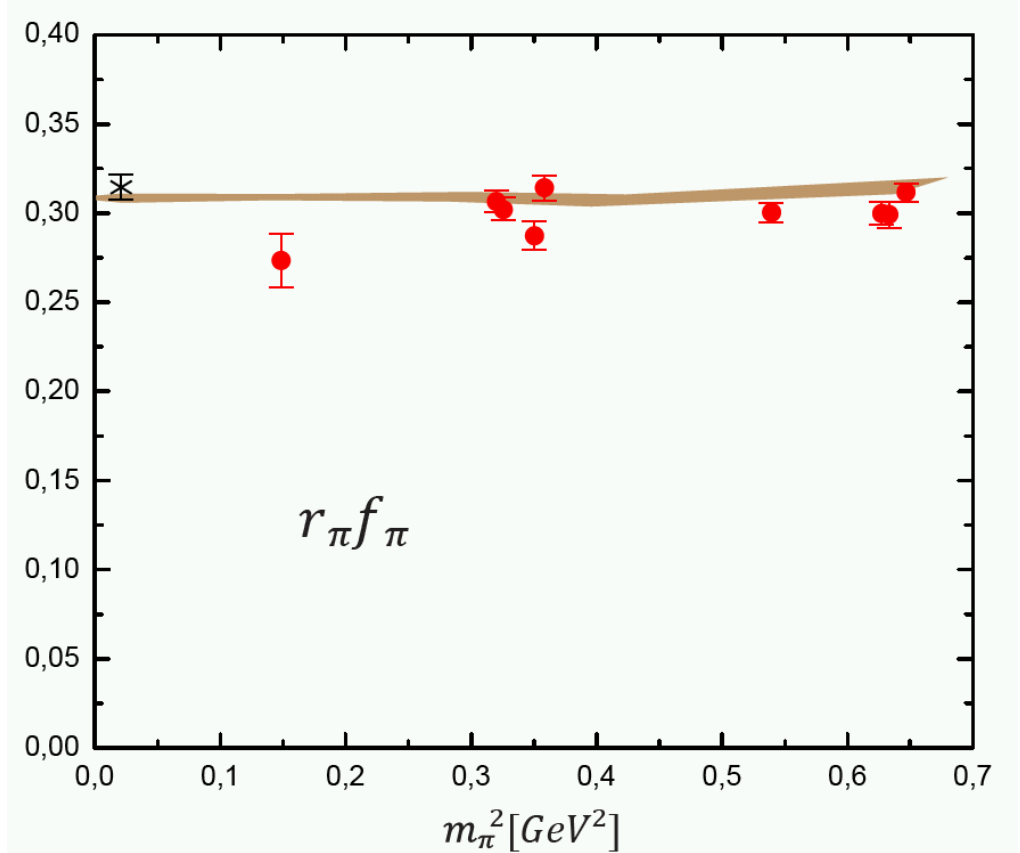
- DSE prediction

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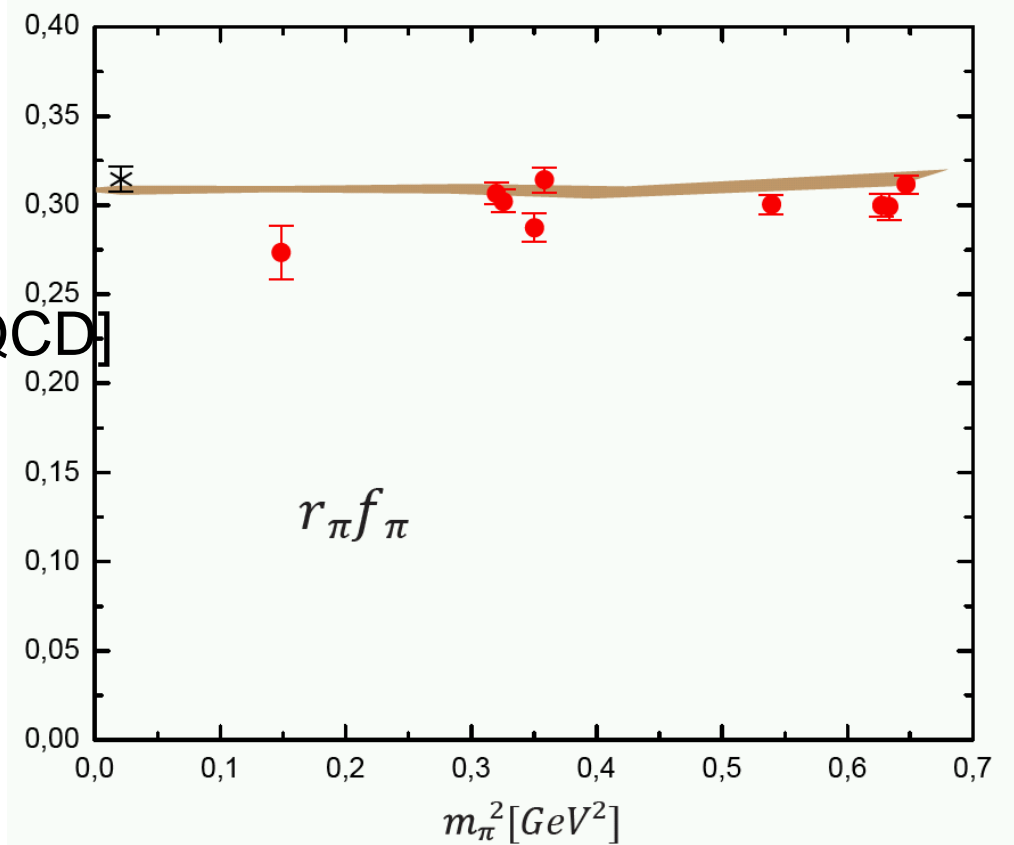
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$r_\pi f_\pi$

$m_\pi^2 [GeV^2]$
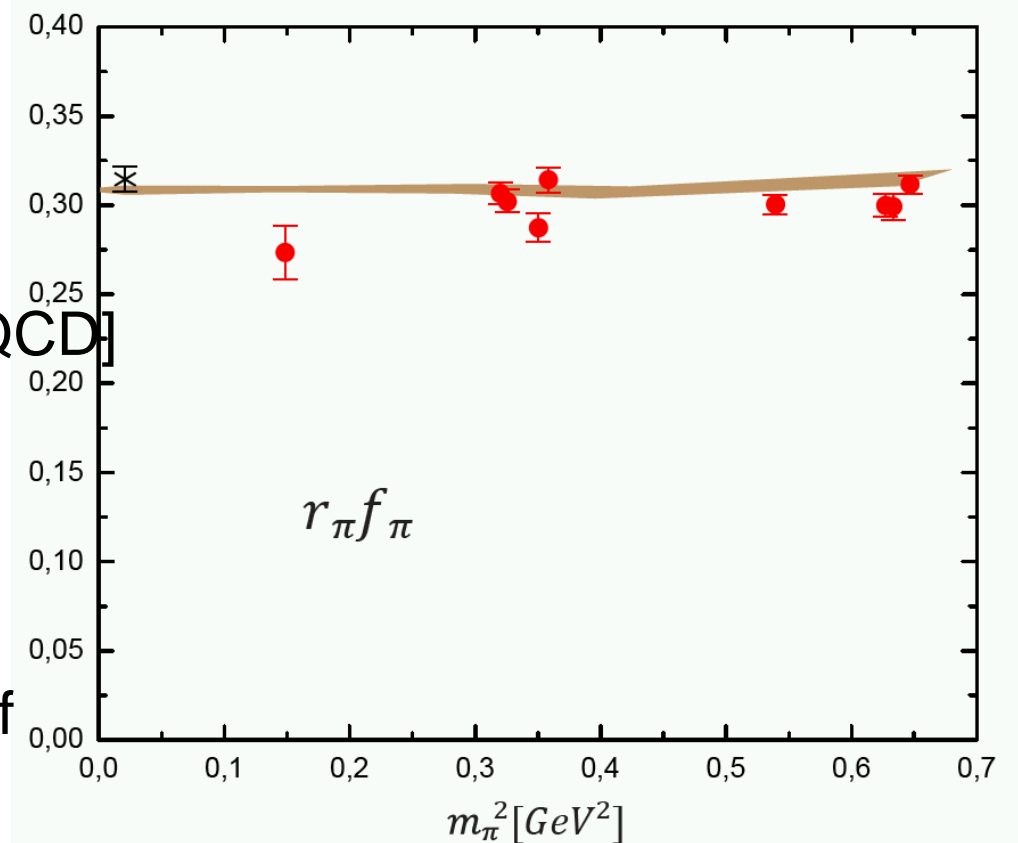
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- We have understood this
  Implications far-reaching.
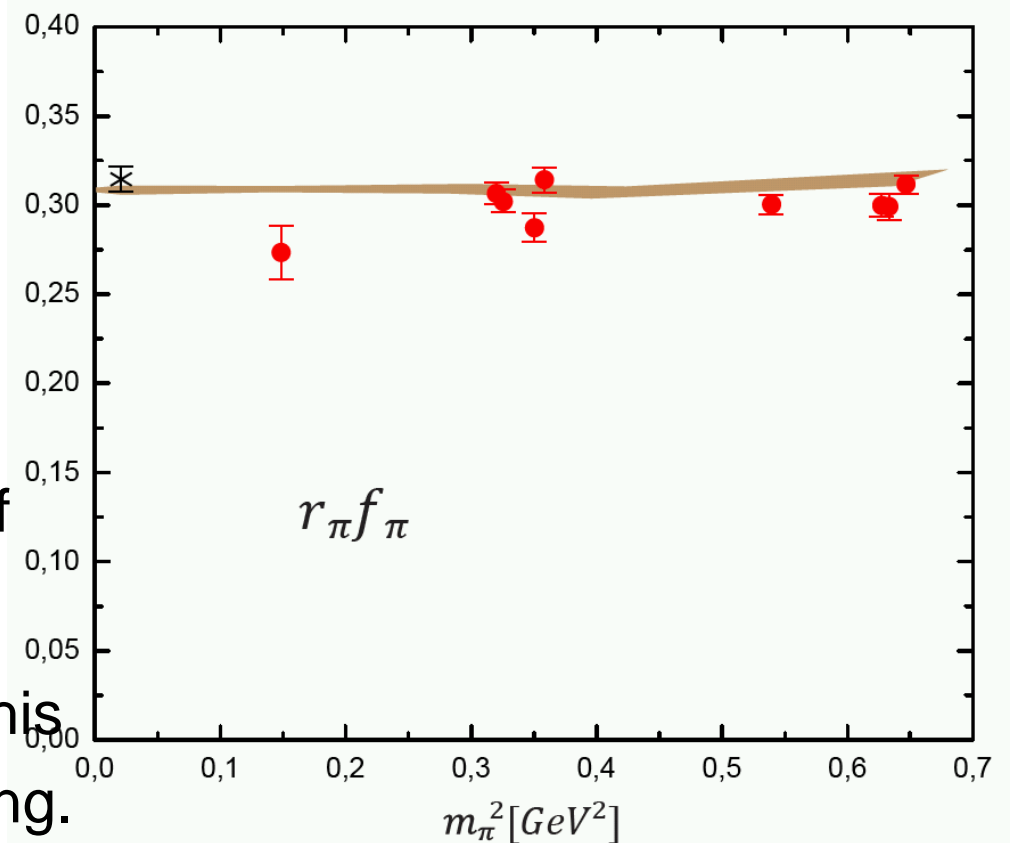
Plot of $r_\pi f_\pi$ versus $m_\pi^2 [GeV^2]$

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New Challenges

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- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

- Another Direction . . . Also want/need information about three-quark systems

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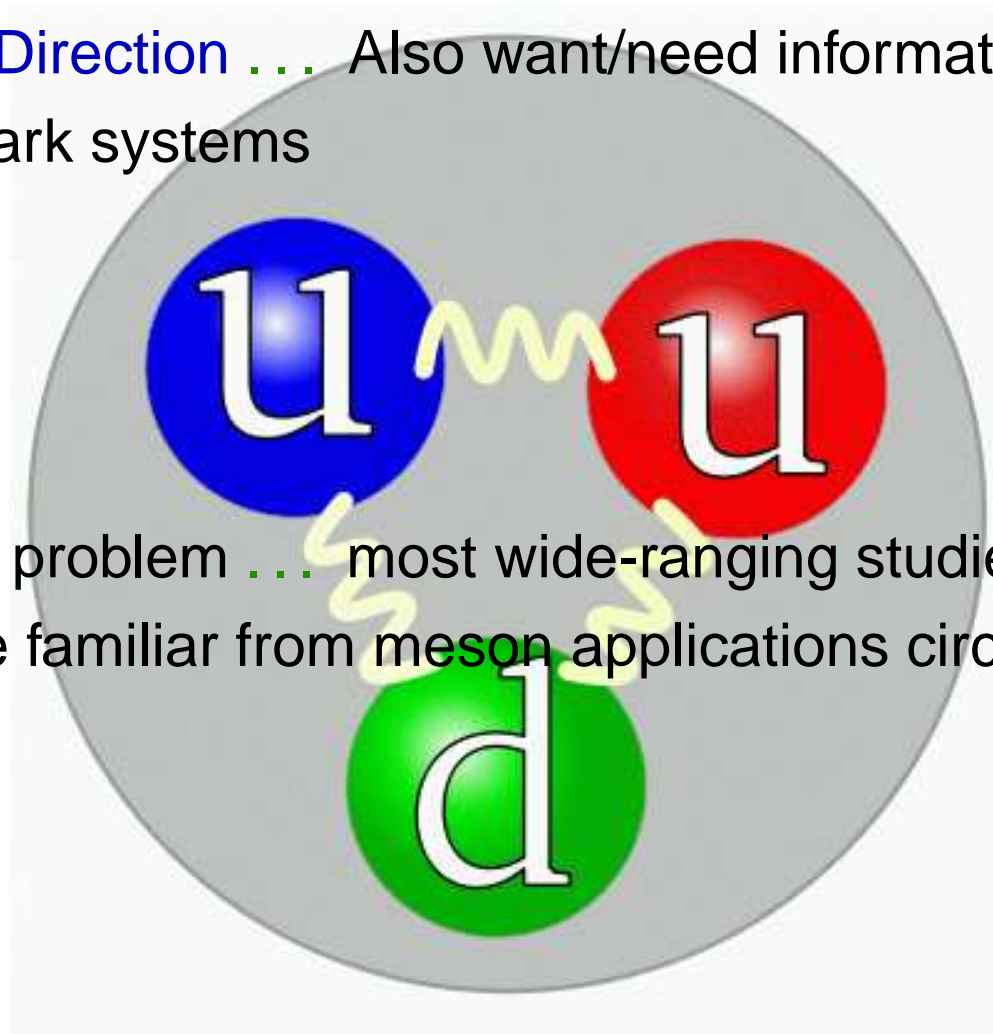
New Challenges

- **Another Direction** ... Also want/need information about three-quark systems

- **With this problem** ... most wide-ranging studies employ expertise familiar from meson applications circa ~ 1995.

- **Namely** ... Model-building and Phenomenology, constrained by the DSE results outlined already.
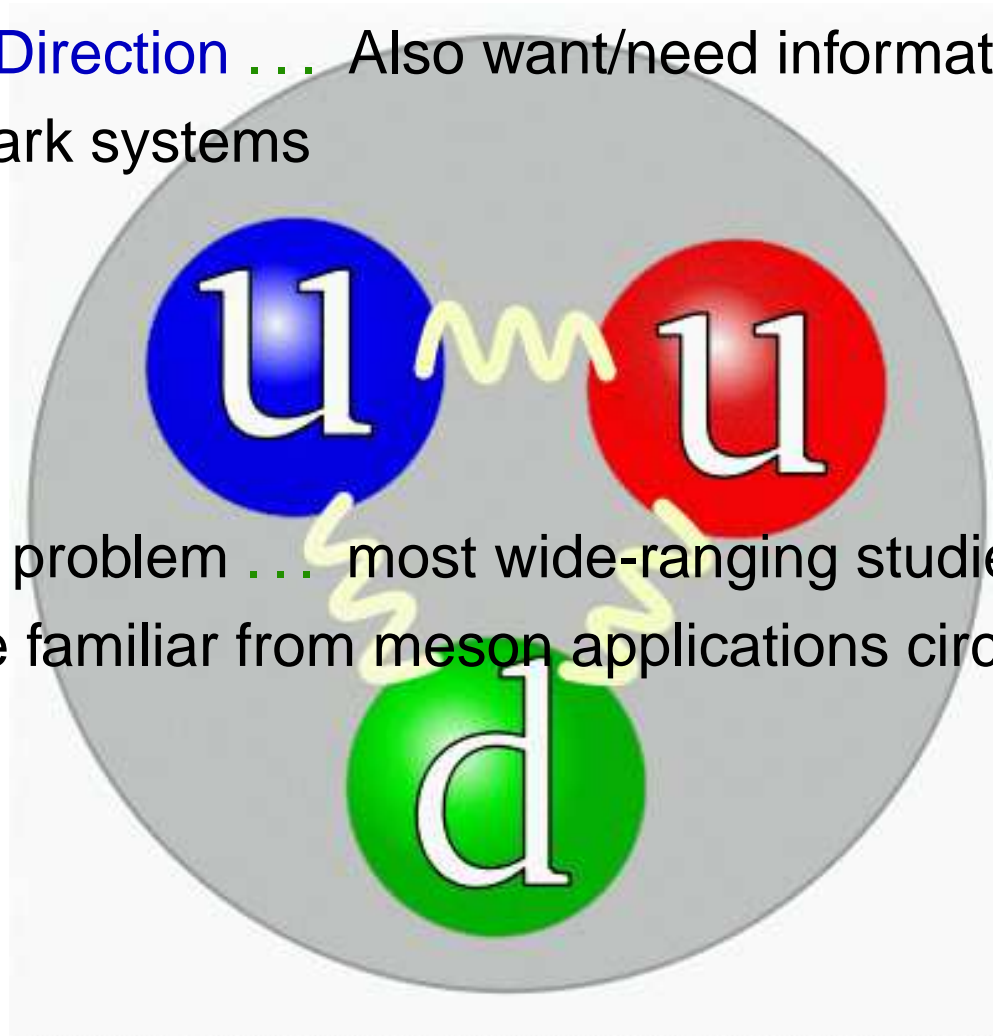
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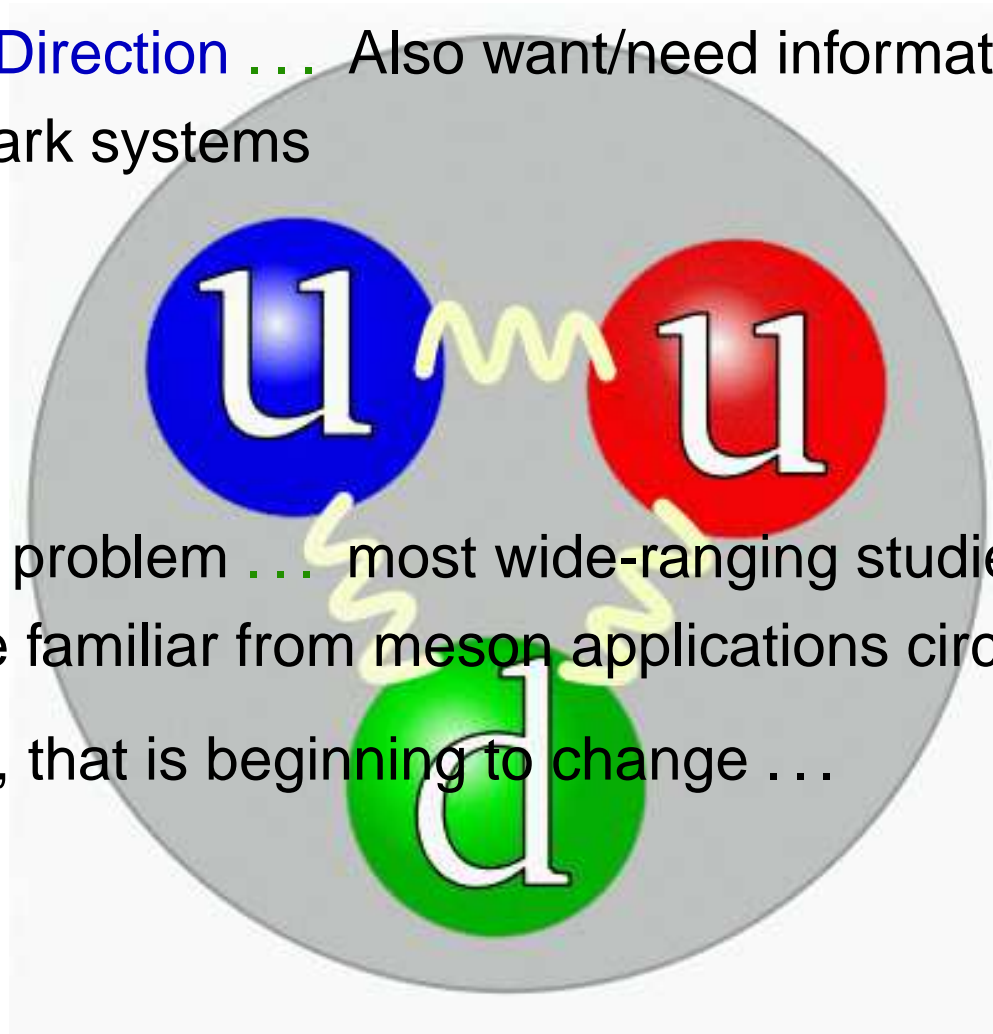
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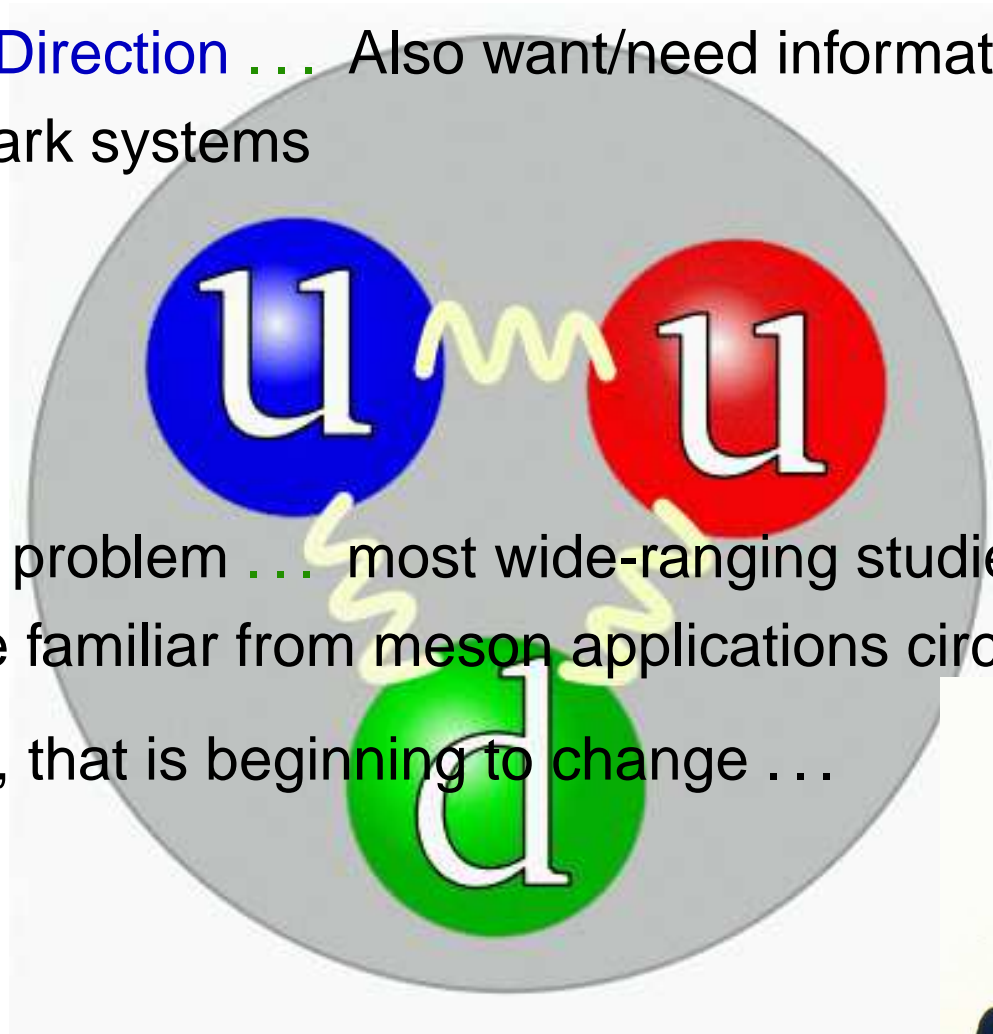
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What is the picture in quantum field theory?

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U.S. DEPARTMENT OF ENERGY

Office of Nuclear Physics
Exploring Nuclear Matter · Quarks to Stars

Argonne
NATIONAL
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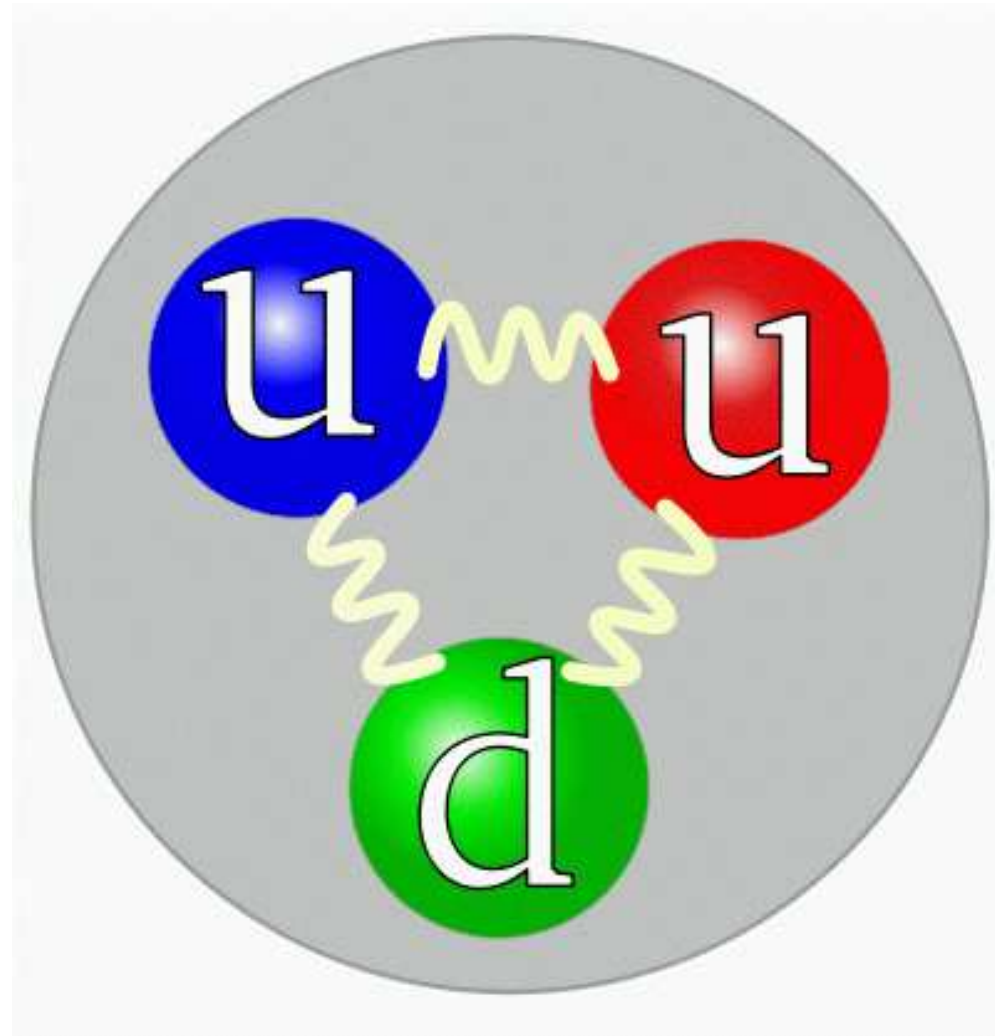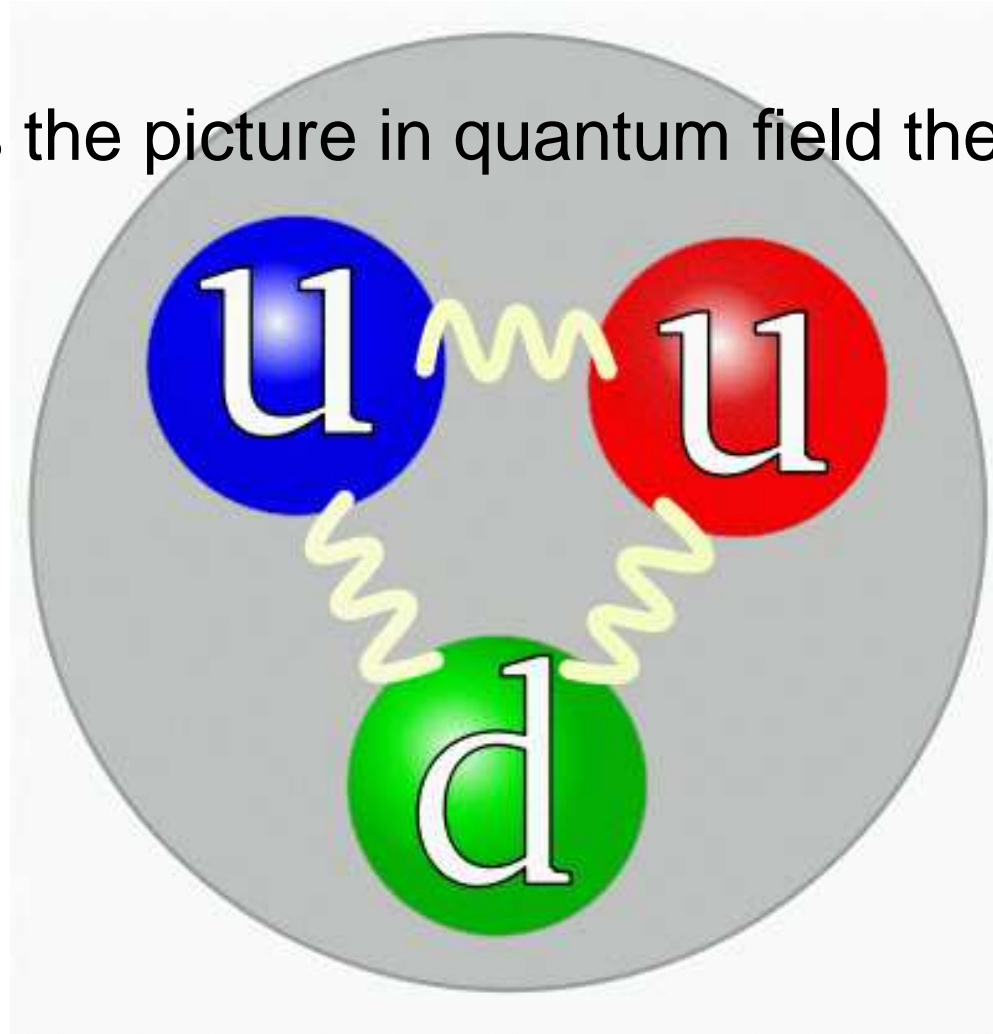
- What is the picture in quantum field theory?
- Three → infinitely many!

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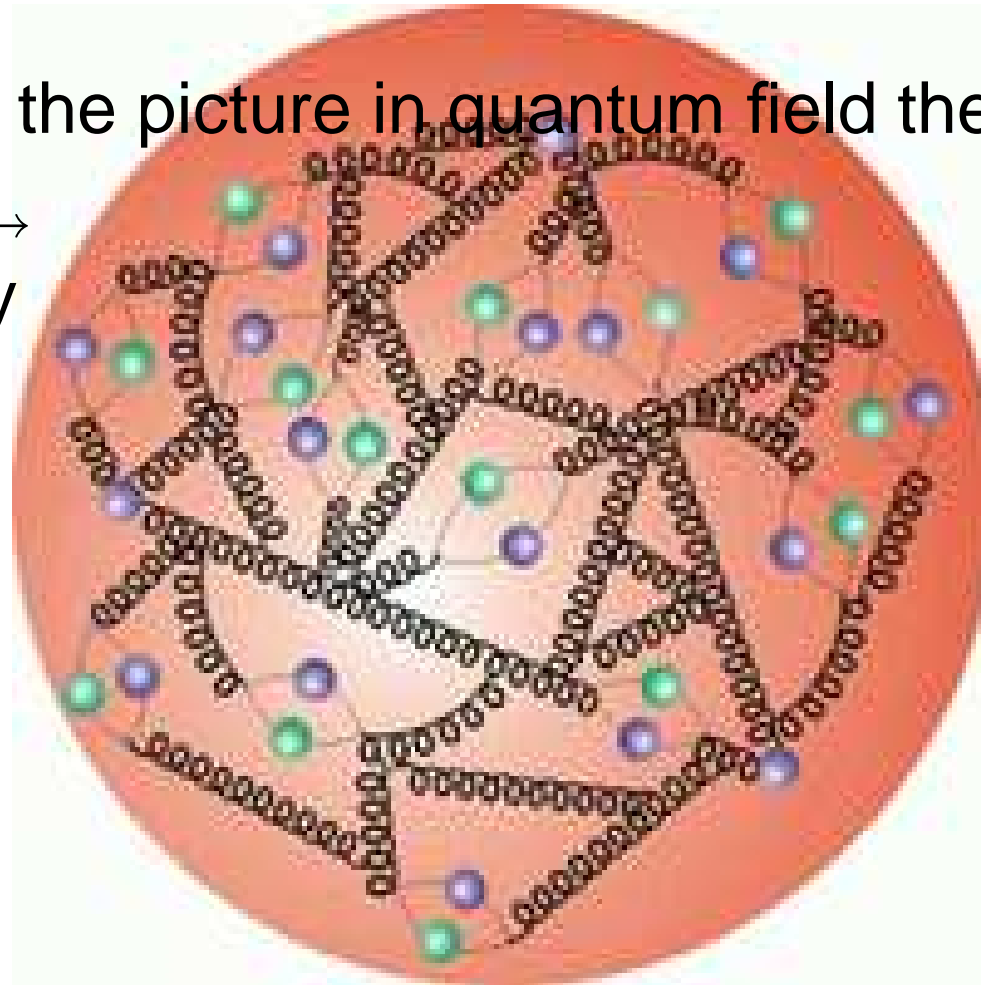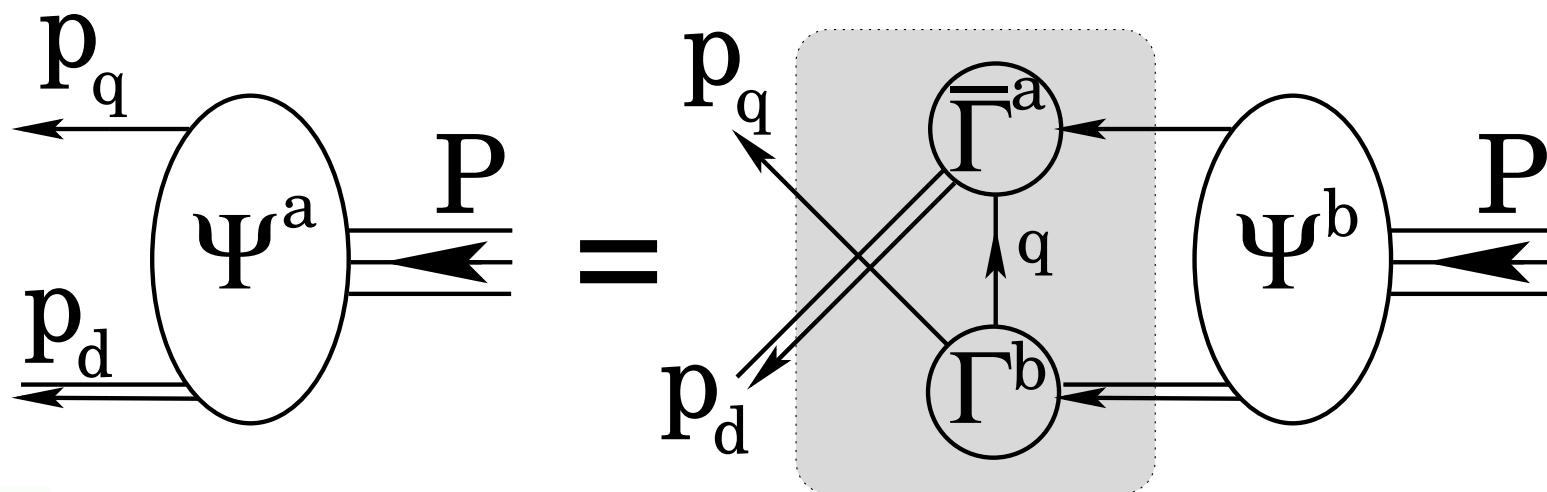
Faddeev equation

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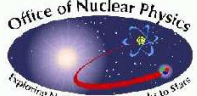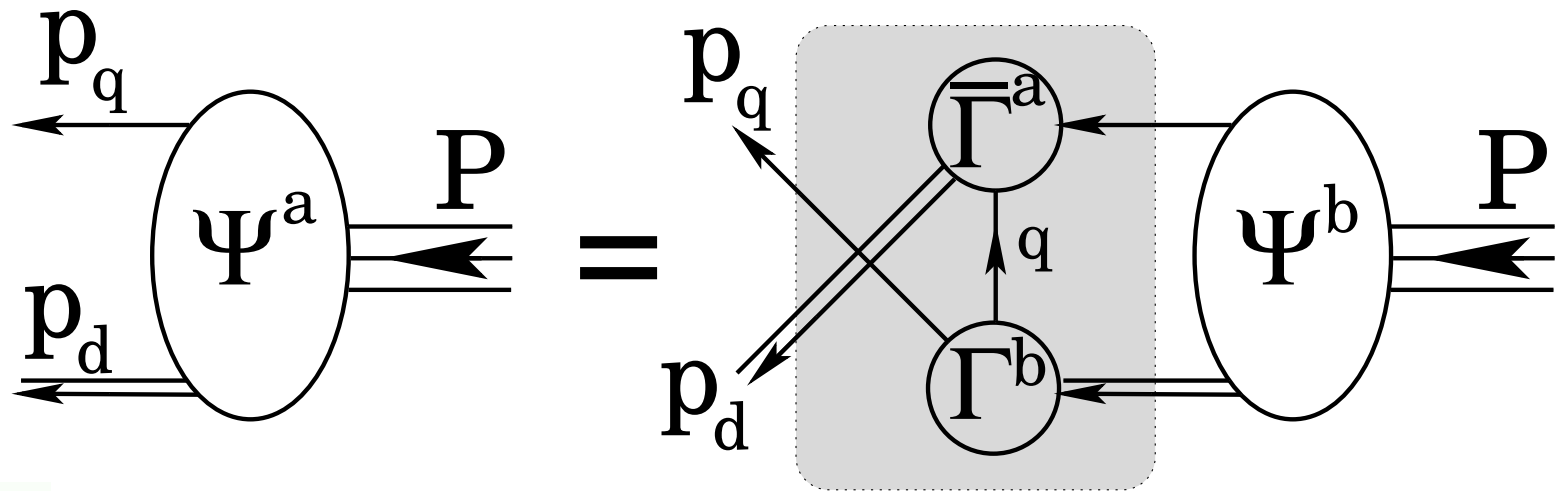
- Linear, Homogeneous Matrix equation

  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations

Diquark correlations
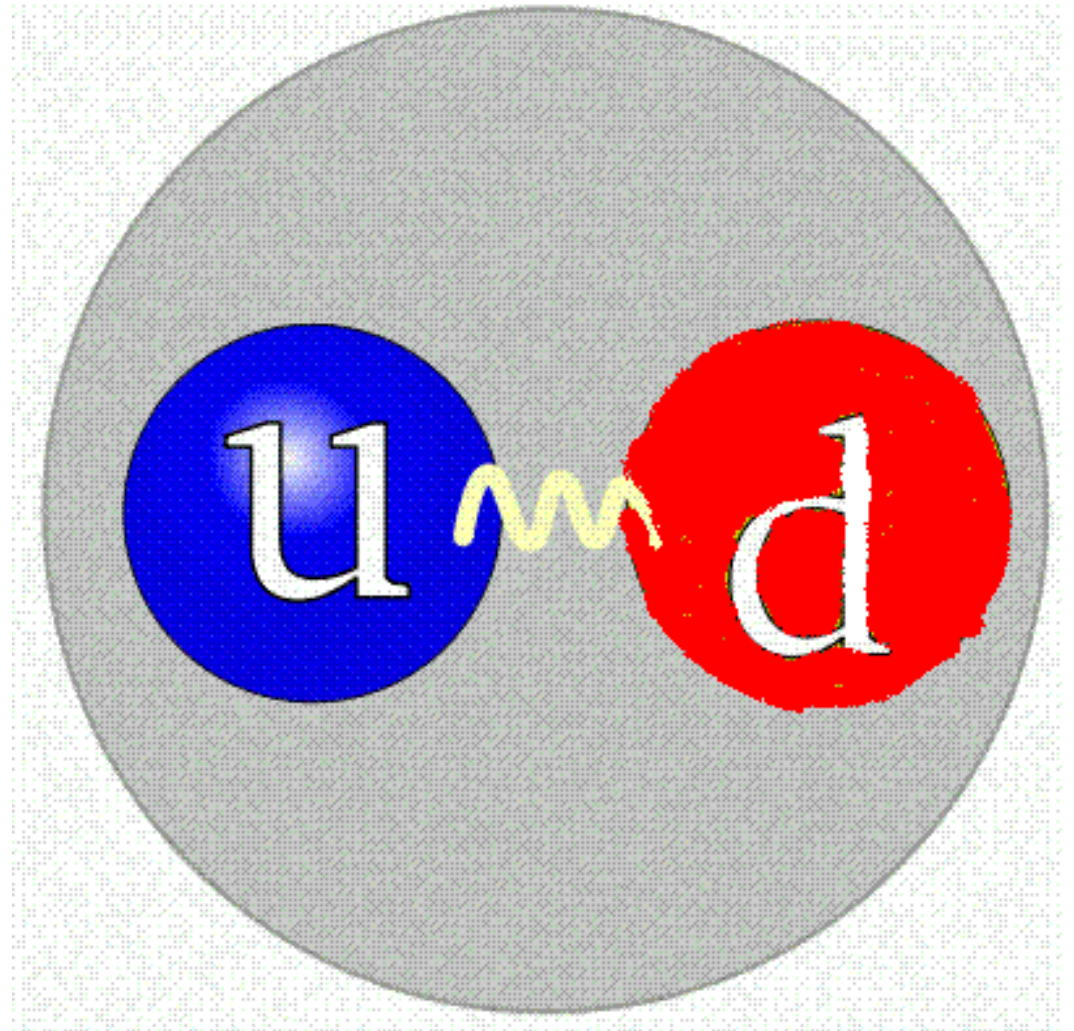
QUARK-QUARK

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red

- Confined ... Does not escape from within baryon.

- Scalar is isosinglet, Axial-vector is isotriplet

- DSE and lattice-QCD

$$m_{[ud]_{0+}} = 0.74 - 0.82$$

$$m_{(uu)_{1+}} = m_{(ud)_{1+}} = m_{(dd)_{1+}} = 0.95 - 1.02$$

QUARK-QUARK

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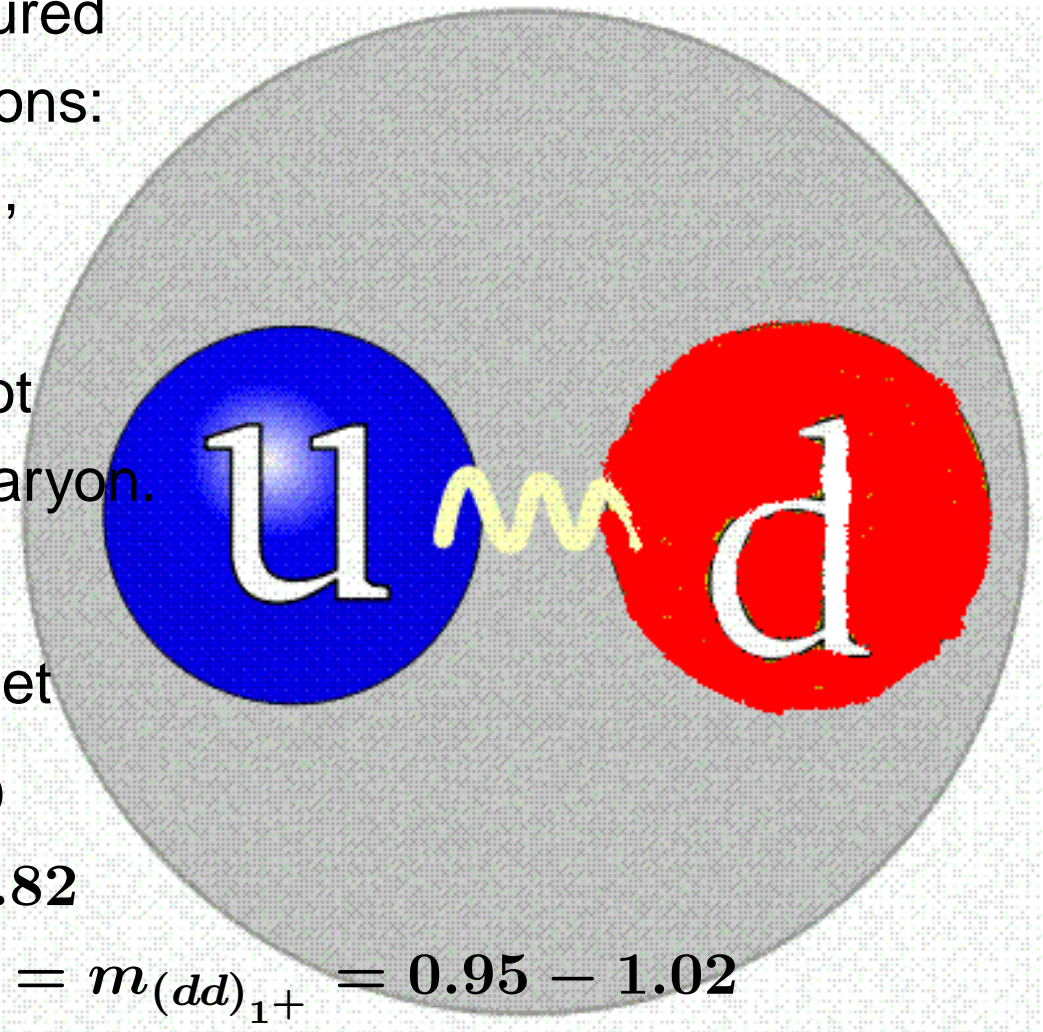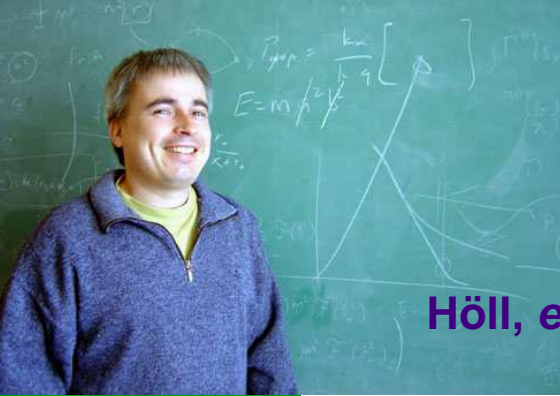
Nucleon EM Form Factors: A Précis

Höll, *et al.*: **nu-th/0412046 & nu-th/0501033**

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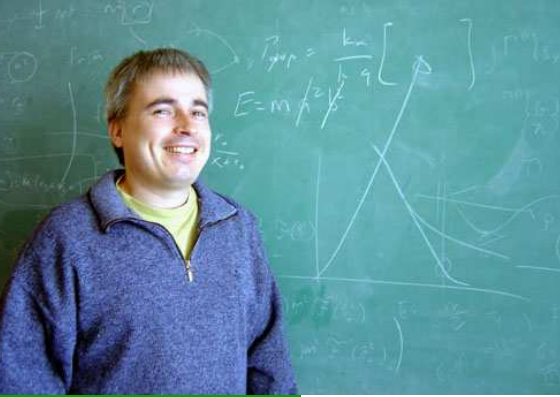
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Nucleon EM Form Factors: A Précis

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Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
 arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

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Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

- Interpreting expts. with GeV electromagnetic probes

  requires Poincaré covariant treatment of baryons

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  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\mathrm{exp}} - M_H^{\mathrm{calc}}]^2}{[M_H^{\mathrm{exp}}]^2} \right)^{1/2} = 2\%$$

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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\mathbf{loop}} = -\mathbf{300}$ to $-\mathbf{400}$ MeV!
- Critical to anticipate pion cloud effects

  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

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Nucleon's self-energy - pion loop

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$$
\begin{aligned}
\Sigma(P) &= 3 \int \frac{d^4 k}{(2\pi)^4} \, g_{PV}^2(P,k) \, \Delta_\pi((P-k)^2) \\
&\quad \times \boxed{\gamma \cdot (P-k)\gamma_5} \, G(k) \, \boxed{\gamma \cdot (P-k)\gamma_5} \\
&= i\gamma \cdot k \, [\mathcal{A}(k^2) - 1] + \mathcal{B}(k^2)
\end{aligned}
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● Pseudovector coupling

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\end{aligned}
$$

- Pseudovector coupling
- Completely equivalent to pseudoscalar coupling
  IF that is treated completely

  - Tadpole contribution can't be neglected

(Hecht, Oettel, Roberts, Schmidt, Tandy, Thomas: nucl-th/0201084)

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- $g_{PV}(P, k)$, πN vertex function

- Calculated using Γ_π and Ψ_N

  - Always soft: Monopole $\lambda \sim 0.6$ GeV

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- $g_{PV}(P,k)$, πN vertex function

- Calculated using Γ_π and Ψ_N

  - Always soft: Monopole $\lambda \sim 0.6$ GeV
    - Corresponds to range $r_\lambda \sim 0.8$ fm
      ... pion cloud does not
      penetrate deeply within nucleon.
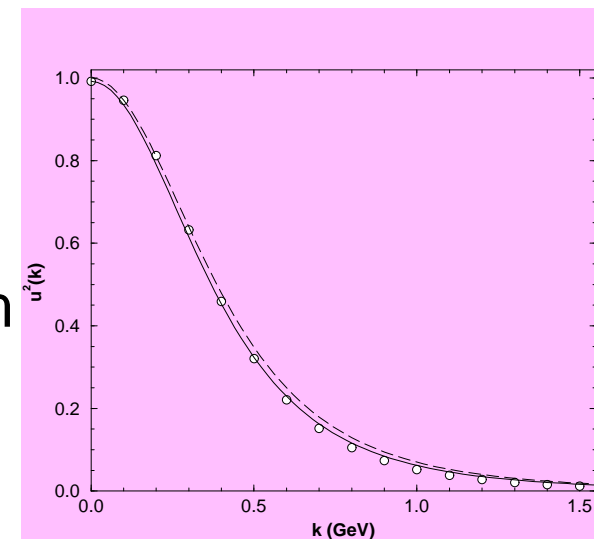
Nucleon's self-energy - pion loop

$$\Sigma(P) \;=\; 3 \int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\, \Delta_\pi((P-k)^2)$$

$$\times \boxed{\gamma \cdot (P-k)\gamma_5}\, G(k) \boxed{\gamma \cdot (P-k)\gamma_5}$$

$$=\; i\gamma \cdot k\, [\mathcal{A}(k^2) - 1] + \mathcal{B}(k^2)$$

$$G(k) \;=\; 1/[i\gamma \cdot k + M + \Sigma(P)] \quad \text{Pole Position Not}$$

$$=\; -i\gamma \cdot k\, \sigma_{\mathcal{V}}(k^2) + \sigma_{\mathcal{S}}(k^2) \quad \text{Known } \textit{a priori}$$

● Mass shift calculated via self-consistent solution

Nucleon's self-energy - pion loop

$$\Sigma(P) \;=\; 3 \int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2 (\text{const.})\, \Delta_\pi((P-k)^2)$$

$$\times \boxed{\gamma \cdot (P-k)\gamma_5}\, G(k)\, \boxed{\gamma \cdot (P-k)\gamma_5}$$

$$=\; i\gamma \cdot k\,[\mathcal{A}(k^2) - 1] + \mathcal{B}(k^2)$$

- Obtain Integral Equation Kernels

$$\int d\Omega_k\, f((P-k)^2) = \frac{2}{\pi} \int_{-1}^{1} dz\, \sqrt{1-z^2}\, f(P^2 + k^2 - 2Pkz)$$

E.g.

$$\omega_{\mathcal{B}}(P^2, k^2) \;=\; \int d\Omega_k\, \frac{(P-k)^2}{(P-k)^2 + m_\pi^2} = 1 - \frac{2\,m_\pi^2}{a + \sqrt{a^2 - b^2}},$$

$$a = P^2 + k^2 + m_\pi^2,\; \mathsf{b} = 2Pk$$

Nucleon's self-energy - pion loop

$$\Sigma(P) = 3 \int \frac{d^4 k}{(2\pi)^4} \, g_{PV}^2(P,k) \, \Delta_\pi((P-k)^2)$$

$$\times \boxed{\gamma \cdot (P-k)\gamma_5} \, G(k) \, \boxed{\gamma \cdot (P-k)\gamma_5}$$

$$= i\gamma \cdot k \, [\mathcal{A}(k^2) - 1] + \mathcal{B}(k^2)$$

- But $g_{PV} = g_{PV}(P^2, k^2, P \cdot k)$

  Therefore, In General, Kernel only known Numerically

- Complicates analysis . . .

  locating, incorporating poles in integrand

Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

Nucleon Self Energy: Chiral Limit

- Let's look what happens when $m_\pi \to 0$
  - Minkowski Space
  - Pseudovector Coupling

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- One-loop nucleon self energy

$$\Sigma(P) = 3i\frac{g^2}{4M^2} \int \frac{d^4k}{(2\pi)^4} \Delta(k^2, m_\pi^2) \, \slashed{k} \, \gamma_5 \, G_0(P-k) \, \slashed{k} \, \gamma_5 \, .$$

This integral is divergent. Assume a Poincaré covariant regularisation, characterised by a mass-scale λ

Nucleon Self Energy: Chiral Limit

- Let's look what happens when $m_\pi \to 0$

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This integral is divergent. Assume a Poincaré covariant regularisation, characterised by a mass-scale λ

- Decompose nucleon propagator into positive and negative energy components

$$
\begin{aligned}
G_0(P) &= \frac{1}{\not{P} - M_0} = G_0^+(P) + G_0^-(P) \\
&= \frac{M}{\omega_N(\vec{P})} \left[\Lambda_+(\vec{P}) \frac{1}{P_0 - \omega_N(\vec{P}) + i\varepsilon} + \Lambda_-(\vec{P}) \frac{1}{P_0 + \omega_N(\vec{P}) - i\varepsilon} \right] \quad (4)
\end{aligned}
$$

$$\omega_N^2(\vec{P}) = \vec{P}^2 + M^2, \text{ and } \Lambda_\pm(\vec{P}) = (\tilde{\not{P}} \pm M)/(2M), \, \tilde{P} = (\omega(\vec{P}), \vec{P})$$

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- Shift in the mass of a positive energy nucleon nucleon:

$$\delta M_+ = \tfrac{1}{2}\mathrm{tr}_D\left[\Lambda_+(\vec{P}=0)\,\Sigma(P_0 = M, \vec{P}=0)\right]$$

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- Focus on positive energy nucleon's contribution to the loop integral; i.e.,
  $\Delta(k) \, G^+(P-k)$, which we denote: $\delta_F M_+^+$

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  $\Delta(k)\, G^+(P-k)$, which we denote: $\delta_F M_+^+$

  - To evaluate k_0 integral, close contour in lower half-plane, thereby encircling only the positive-energy pion pole.

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3}\, \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)}\, \frac{1}{\omega_\pi(\vec{k}^2)\left[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0\right]} \quad (9)$$

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🔴 On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \quad (15)$$

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🔴 On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \quad (19)$$

🔴 Then

$$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3 k}{(2\pi)^3} \frac{\vec{k}^2}{8\,M^2} \frac{1}{\omega_{\lambda_i}^2(\vec{k}^2)} \quad (20)$$

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- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{23}$$

- Then

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- So that

$$\frac{d^2\,\delta_F M_+^+}{(dm_\pi^2)^2} \approx -\frac{3g^2}{4M^2} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{\omega_\pi^6(\vec{k}^2)} = -\frac{9}{128\pi} \frac{g^2}{M^2} \frac{1}{m_\pi} \,. \tag{25}$$
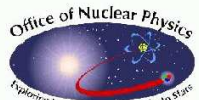
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Nucleon Self Energy: Chiral Limit

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🔴 Namely $\delta_F M_+^+ = -\dfrac{3}{32\pi} \dfrac{g^2}{M^2} m_\pi^3 + f_{(1)}^+(\lambda_1,\lambda_2)\,m_\pi^2 + f_{(0)}^+(\lambda_1,\lambda_2)$

where the last two terms express the necessary contribution from the regulator.

Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.

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  - This is the Leading Nonanalytic Contribution much touted in effective field theory.

  - Its form is completely fixed by chiral symmetry and the pattern of its dynamical breaking.

  NB. Contribution from negative energy nucleon is $\propto \dfrac{1}{M^3}$.

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- The remaining terms are regular in the current-quark mass. Their exact nature depends on the explicit form of regularisation procedure.

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.

- The Leading Nonanalytic Contribution is a model-independent result.

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- The Leading Nonanalytic Contribution is a model-independent result.

- Unfortunately, it is of limited relevance. In a calculation of the nucleon's mass, the actual value of the pion loop's contribution is almost completely determined by the regularisation dependent terms.

  - It is essential for a framework to veraciously express the leading nonanalytic contribution ... it serves as a check that DCSB is truly described.

  - However, beyond that, one must accept that the world is complex.
    - The pion has a finite size. So does the nucleon.
    - These sizes set the mass-scale which determines the nucleon's mass shift.

Model pion-nucleon coupling

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$$g_{PV}(P, k) = \frac{g}{2M} \exp(-(P - k)^2/\Lambda^2)$$

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- $\mathcal{B}$-Kernel

$$\int d\Omega_k \, g_{PV}^2((P-k)^2) \left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2} \right]$$

Clearly the sum of two independent terms.

Model pion-nucleon coupling

$$g_{PV}(P, k) = \frac{g}{2M} \exp(-(P - k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k \, g_{PV}^2((P - k)^2) \left[1 - \frac{2m_\pi^2}{(P - k)^2 + m_\pi^2} \right]$$

- First term can be evaluated exactly

$$\bar{g}_{PV}^2(P^2, k^2) = \int d\Omega_k \, g_{PV}^2((P - k)^2)$$

$$= \frac{g^2}{4M^2} \, e^{-2(P^2 + k^2)/\Lambda^2} \, \frac{\Lambda^2}{2Pk} \, I_1(4Pk/\Lambda^2),$$

Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M} \exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k \, g_{PV}^2((P-k)^2) \left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2}\right]$$

- Second term can be approximated

$$
\begin{aligned}
\omega_{g^2}(P^2, k^2) &= 2\, m_\pi^2 \int d\Omega_k \, \frac{g_{PV}^2((P-k)^2)}{(P-k)^2 + m_\pi^2} \\
&\approx g_{PV}^2(|P-k|^2) \frac{2\, m_\pi^2}{a + \sqrt{a^2 - b^2}}
\end{aligned}
$$

- Reliable when analytic

structure of g_{PV} is not key to that of solution

Model pion-nucleon coupling

$$g_{PV}(P, k) = \frac{g}{2M} \, \exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k \, g_{PV}^2((P-k)^2) \left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2} \right]$$

- Total Kernel:

$$
\begin{aligned}
&\approx \quad \bar{g}_{PV}^2(P^2, k^2) - g_{PV}^2(|P^2 - k^2|) \frac{2\,m_\pi^2}{a + \sqrt{a^2 - b^2}} \,, \\
&=: \quad \bar{g}_{PV}^2(P^2, k^2) - \tilde{g}_{PV}^2(P^2, k^2)) \frac{2\,m_\pi^2}{a + \sqrt{a^2 - b^2}} \,,
\end{aligned}
$$

- Analytic structure is transparent

Nucleon's self energy and mass shift

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$

$$\delta M = M_D - M$$

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$
\begin{aligned}
M_D^2 \mathcal{A}^2(-M_D^2) &= [M + \mathcal{B}(-M_D^2)]^2 \\
\delta M &= M_D - M
\end{aligned}
$$

- Vector self energy

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$
$$\delta M = M_D - M$$

- Scalar self energy

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) \;=\; [M + \mathcal{B}(-M_D^2)]^2$$
$$\delta M \;=\; M_D - M$$

	(Λ, Λ_N)	(Λ, Λ_N)	(Λ, Λ_N)
	$(0.9, \infty)$	$(0.9, 1.5)$	$(0.9, 2.0)$
$-\delta M$ (MeV)	222	61	99

Nucleon's self energy and mass shift

● Solve DSE Nonperturbatively

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\begin{aligned}
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$-\delta M$ (MeV)	222	61	99

● No suppression for nucleon off-shell in self-energy loop;

i.e, $g_{PV}((P - k^2), P^2, k^2)$

Neglected this dependence

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$
\begin{aligned}
M_D^2 \mathcal{A}^2(-M_D^2) &= [M + \mathcal{B}(-M_D^2)]^2 \\
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	$(0.9, \infty)$	$(0.9, 1.5)$	$(0.9, 2.0)$
$-\delta M$ (MeV)	222	61	99

$$
g_{PV}(P^2, k^2, P \cdot k) = \frac{g}{2M}\, e^{-(P-k)^2/\Lambda^2}\, e^{-(P^2+M^2+k^2+M^2)/\Lambda_N^2}
$$

- Correct on-shell limit:

$$
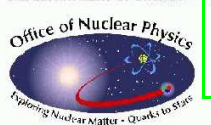g_{PV}(P^2 = -M^2, k^2 = -M^2, (P-k)^2 = 0) = \frac{g}{2M}
$$

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$

$$\delta M = M_D - M$$

Range from meson exchange model phen.

	(Λ, Λ_N)	(Λ, Λ_N)	(Λ, Λ_N)
	$(0.9, \infty)$	$(0.9, 1.5)$	$(0.9, 2.0)$
$-\delta M$ (MeV)	222	61	99

$$g_{PV}(P^2, k^2, P \cdot k) = \frac{g}{2M} \, e^{-(P-k)^2/\Lambda^2} \, e^{-(P^2+M^2+k^2+M^2)/\Lambda_N^2}$$

- $\Lambda_N \to \infty \Rightarrow$ pointlike nucleon

- Nonpointlike πN-loop

  ... reduces nucleon's mass by $\sim 100\,\mathrm{MeV}$

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Pion loop's effect

- Nonpointlike πN-loop

  ... reduces nucleon's mass by $\sim 100\,$MeV

- There's also a $\pi\Delta$-loop

  ... reduces nucleon's mass by not more than $100\,$MeV

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  ... reduces nucleon's mass by not more than $100\,\text{MeV}$

$$-\delta M_N \sim 200\,\text{MeV}$$

- Qualitative effect of this?

Too much of a good thing

Too much of a good thing

- Refit Faddeev model parameters,

  allowing for heavier "quark-core" mass

Too much of a good thing

	ω_{0^+}	ω_{1^+}	M_N	M_Δ	ω_{f_1}	ω_{f_2}	R
0^+	0.45	-	1.44	-	0.36	0.35	2.32
0^+ & 1^+	0.45	1.36	1.14	1.33	0.44	0.36	0.54
0^+	0.64	-	1.59	-	0.39	0.41	1.28
0^+ & 1^+	0.64	1.19	0.94	1.23	0.49	0.44	0.25

- 50% reduction in role of axial-vector diquark

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0^+ & 1^+	0.64	1.19	0.94	1.23	0.49	0.44	0.25

- 50% reduction in role of axial-vector diquark

- 10% increase in role of scalar diquark

Too much of a good thing

	ω_{0+}	ω_{1+}	M_N	M_Δ	ω_{f_1}	ω_{f_2}	R
0^+	0.45	-	1.44	-	0.36	0.35	2.32
0^+ & 1^+	0.45	1.36	1.14	1.33	0.44	0.36	0.54
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Unsurprisingly:

Requiring Exact Fit to N, Δ masses

with only q, $(qq)_{JP}$ Degrees of Freedom

$\Rightarrow$ Forces 1^+ to mimic, in part, effect of π

Pseudoscalar mesons and Form Factors

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- Light mass of pseudoscalar mesons means they play a very important role in many aspects of hadron physics.

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- Light mass of pseudoscalar mesons means they play a very important role in many aspects of hadron physics.

- Indeed, no approach to low-energy hadron physics that does not explicitly account for pseudoscalar meson degrees of freedom can be valid.

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Pseudoscalar mesons and Form Factors

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- Another example . . . pseudoscalar mesons also contribute materially to form factors.

- Light mass of pseudoscalar mesons means they play a very important role in many aspects of hadron physics.

- Indeed, no approach to low-energy hadron physics that does not explicitly account for pseudoscalar meson degrees of freedom can be valid.

- Another example ... pseudoscalar mesons also contribute materially to form factors.

- Illustrate with $\gamma N \to \Delta$ transition form factor. Focus on the M1 (spin-flip) form factor, $G_M^*(Q^2)$.

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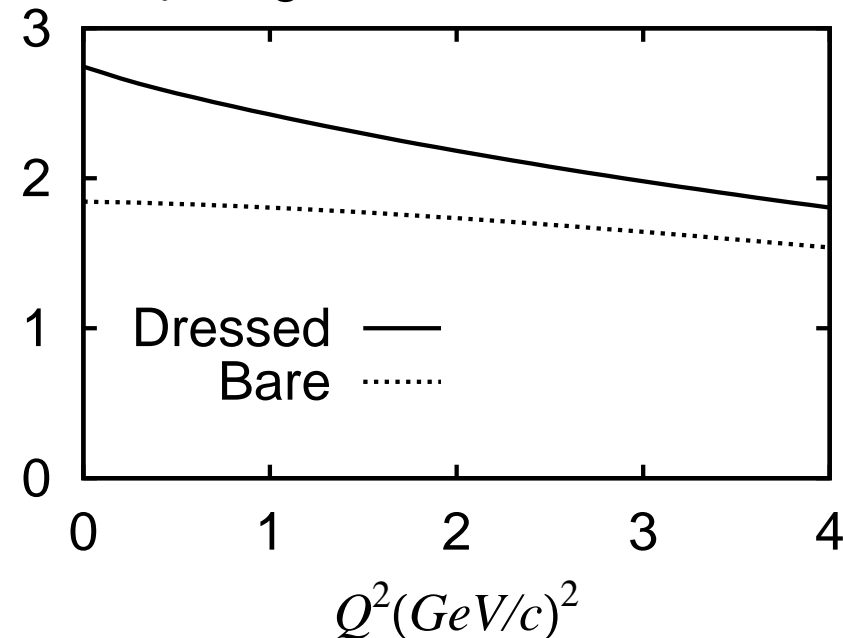
Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$

  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)

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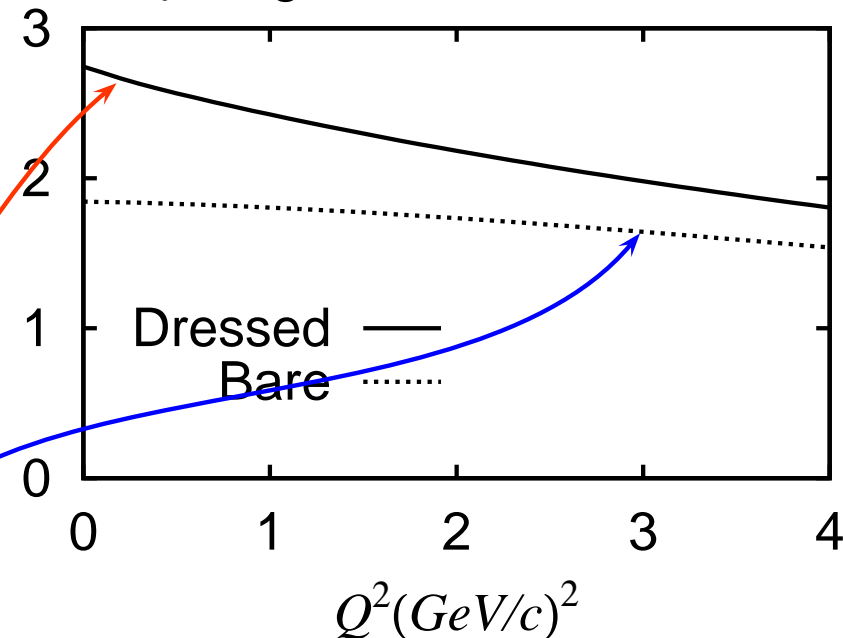
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- Pion cloud effects are large in the low Q^2 region.

Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.

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Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 > 2 - 3\,\mathrm{GeV}^2$

Dressed ——
Bare ········

$Q^2 (GeV/c)^2$

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\triangle$ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	$M_\triangle$	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.80	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

● $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\triangle$ masses

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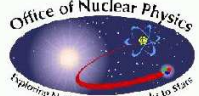
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- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

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Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

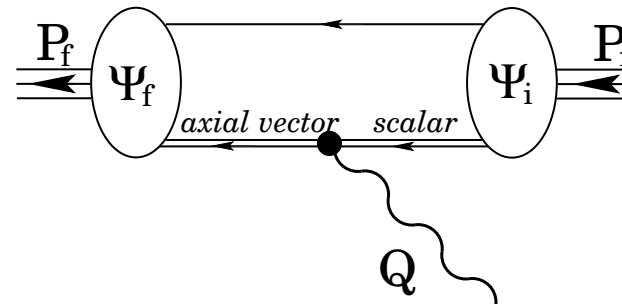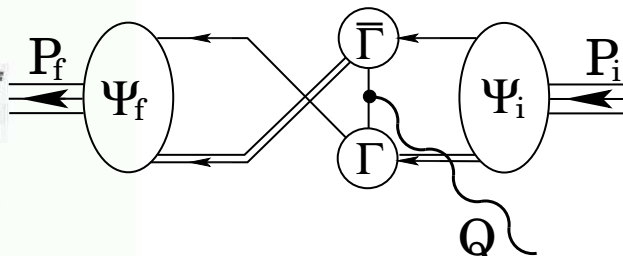# Nucleon-Photon Vertex

6 terms . . .
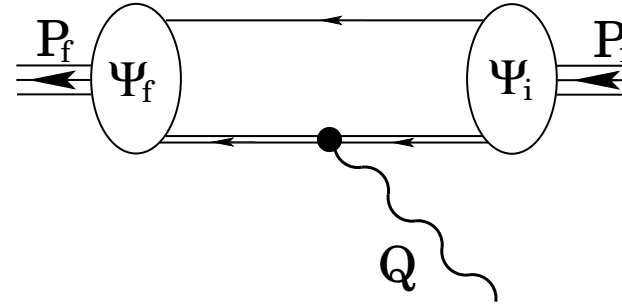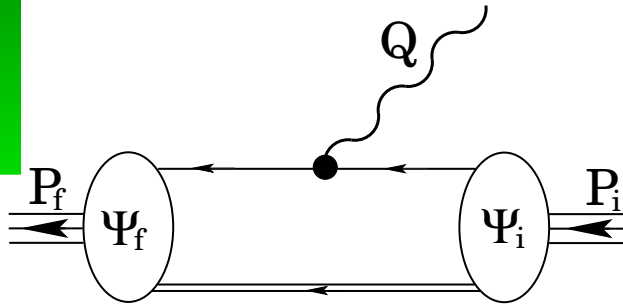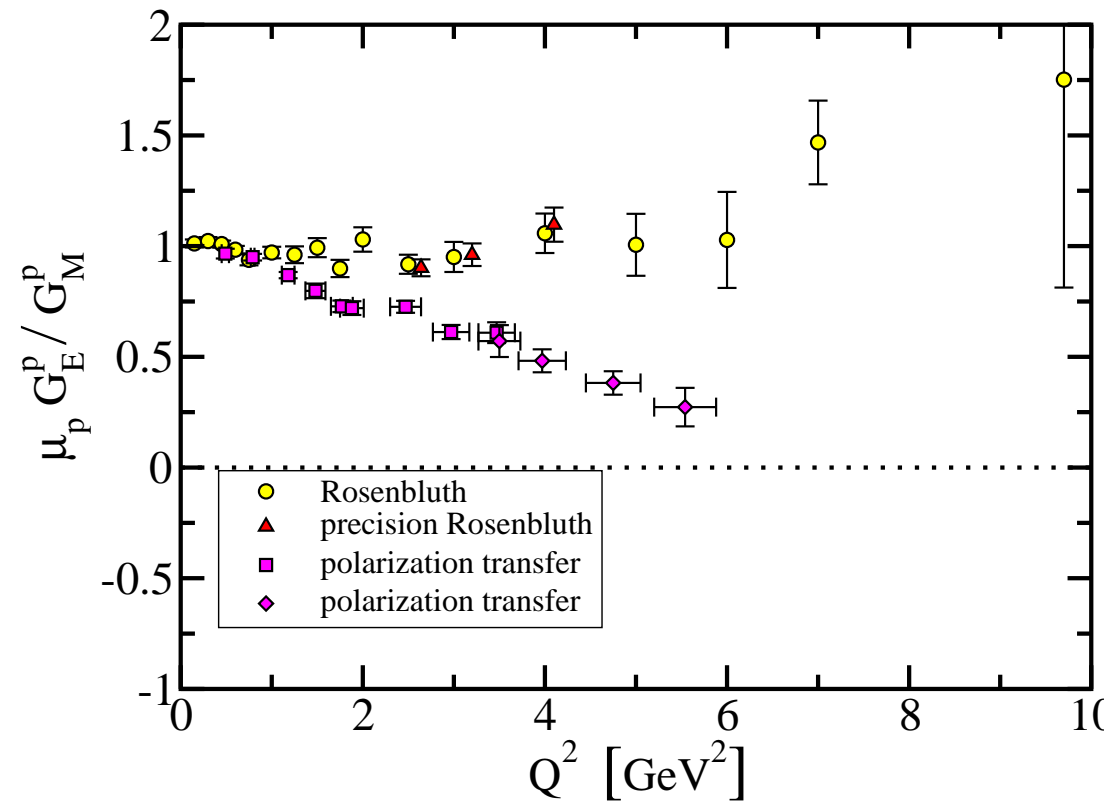
constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

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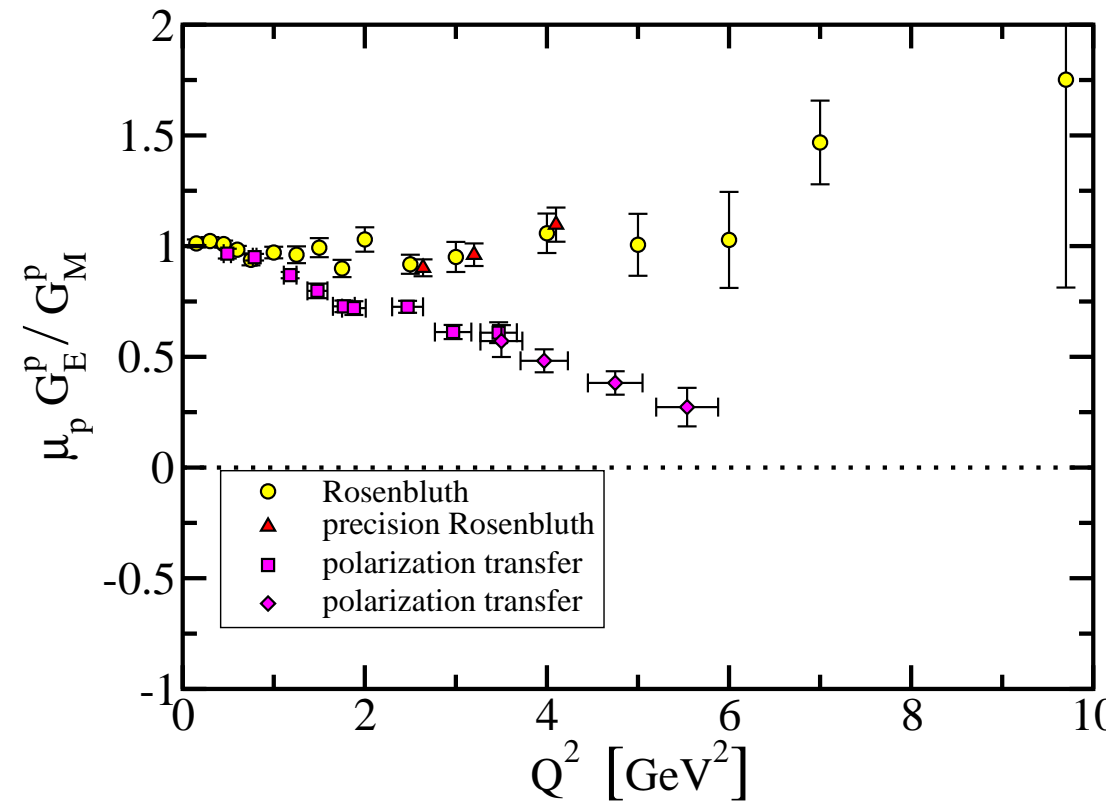
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Combine these elements …

- Combine these elements …

  - Dressed-Quark Core

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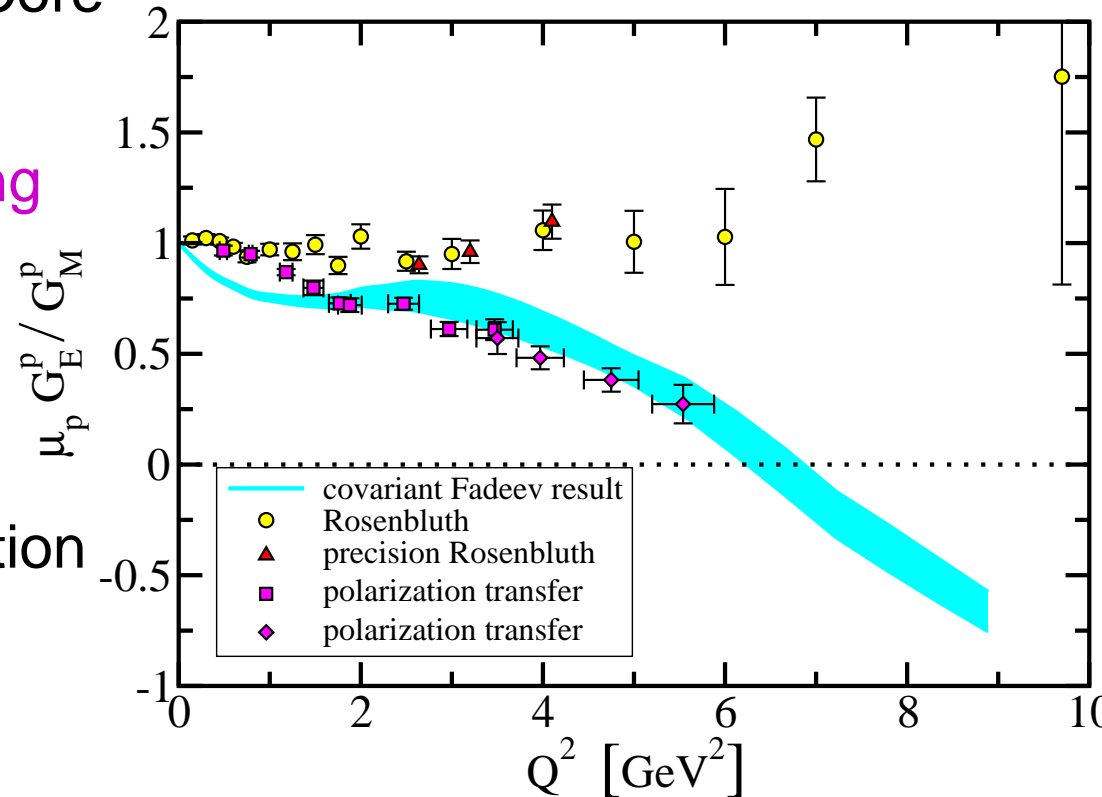
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  - Anticipate and Estimate Pion Cloud's Contribution

Legend within plot:
- ○ Rosenbluth
- ▲ precision Rosenbluth
- ■ polarization transfer
- ◆ polarization transfer

Y-axis: $\mu_p \, G_E^p / G_M^p$

X-axis: $Q^2 \; [\text{GeV}^2]$

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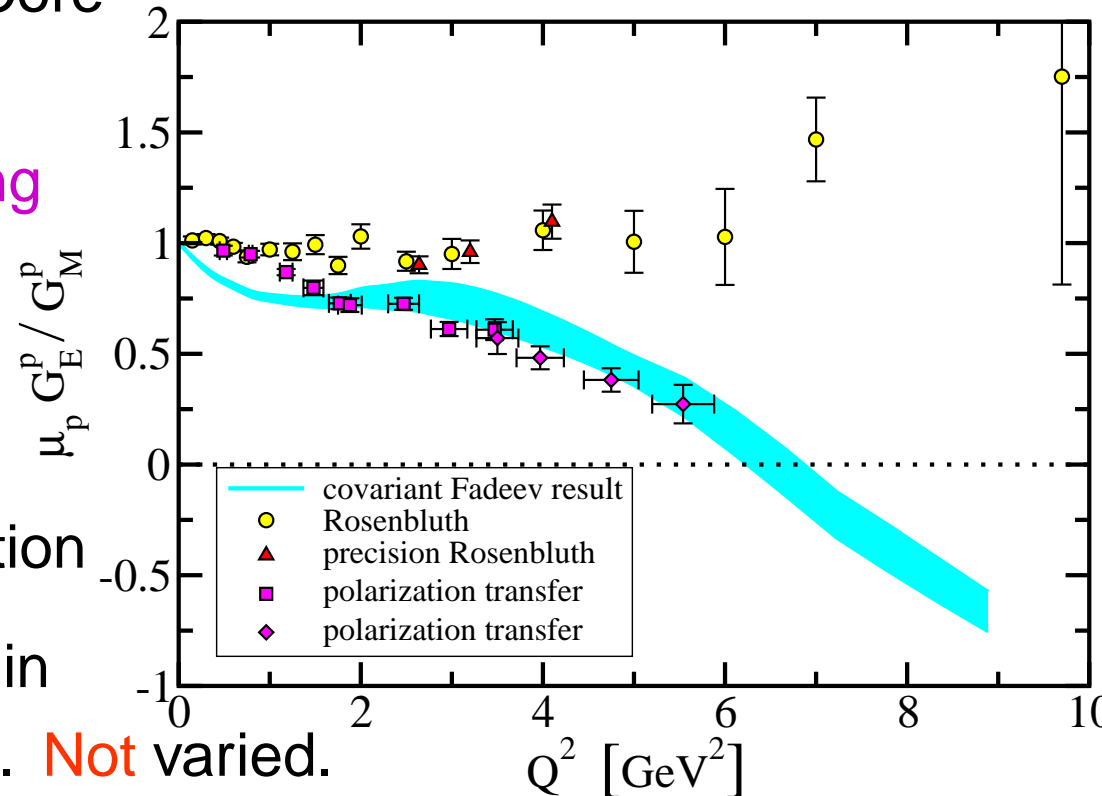
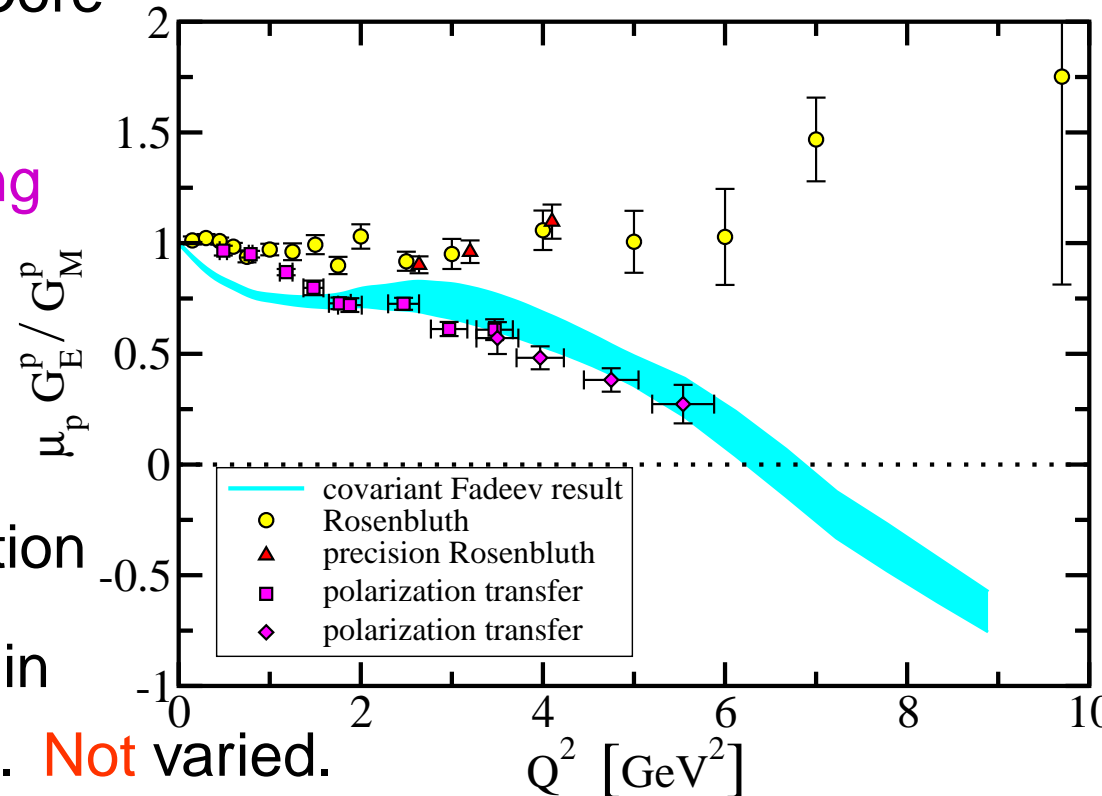- Combine these elements . . .

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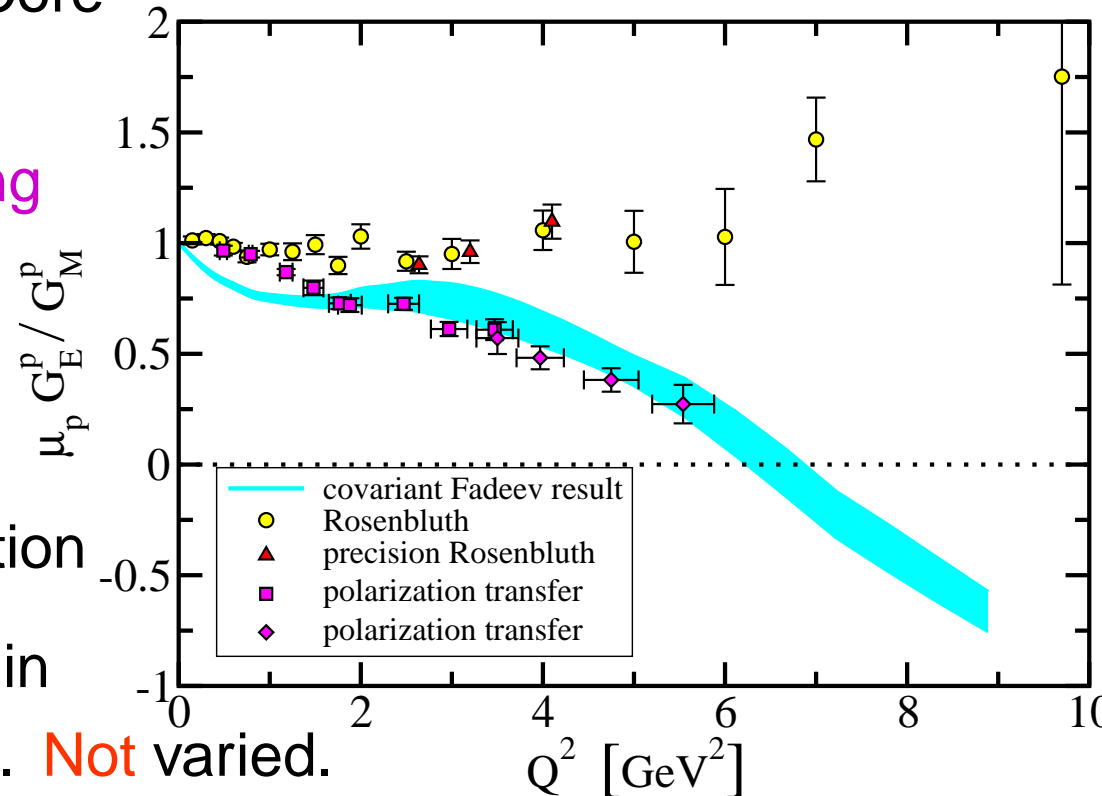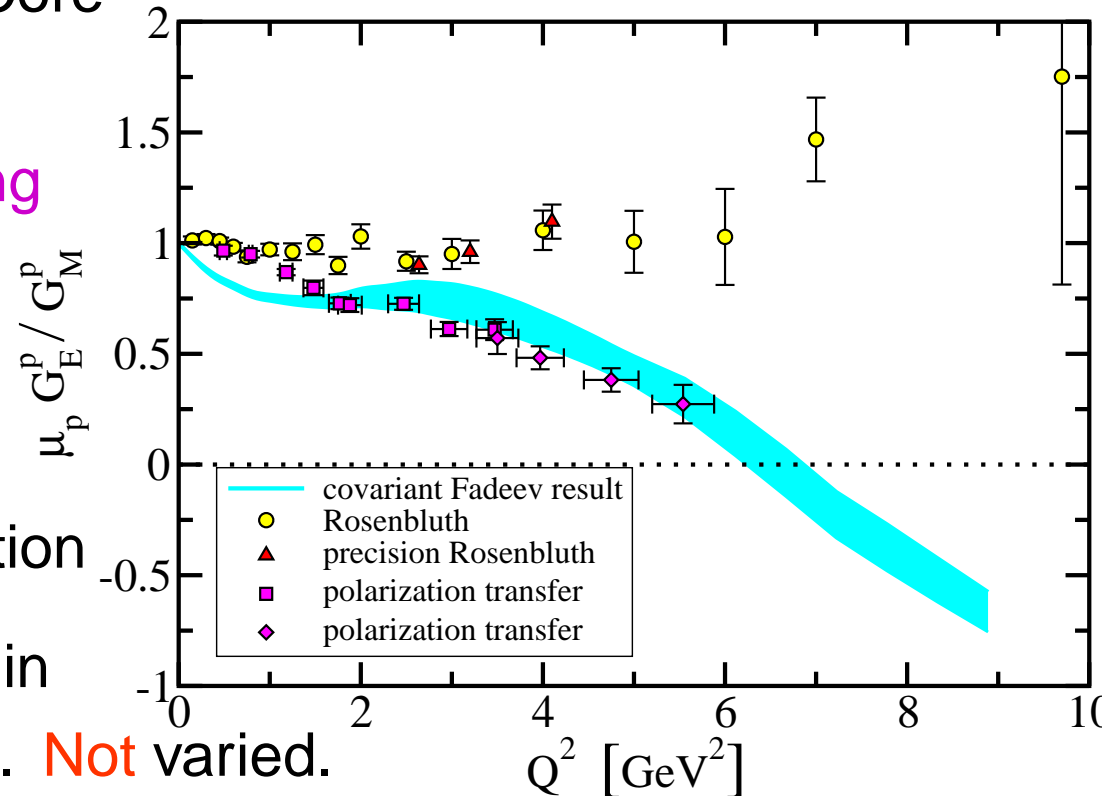
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  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$

Figure axis labels: vertical $\mu_p\, G_E^p / G_M^p$; horizontal Q^2 [GeV^2]

Legend:
- covariant Fadeev result
- Rosenbluth
- precision Rosenbluth
- polarization transfer
- polarization transfer

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  - Predict Zero at $Q^2 \approx 6.5\mathrm{GeV^2}$

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- Proton's Electromagnetic Form Factor

  - Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected

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- Proton's Electromagnetic Form Factor

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Peak Position and Height
-- as zero moves in toward q*q=0
peak moves out and falls in magnitude
-- redistribution of charge ... moves to larger r

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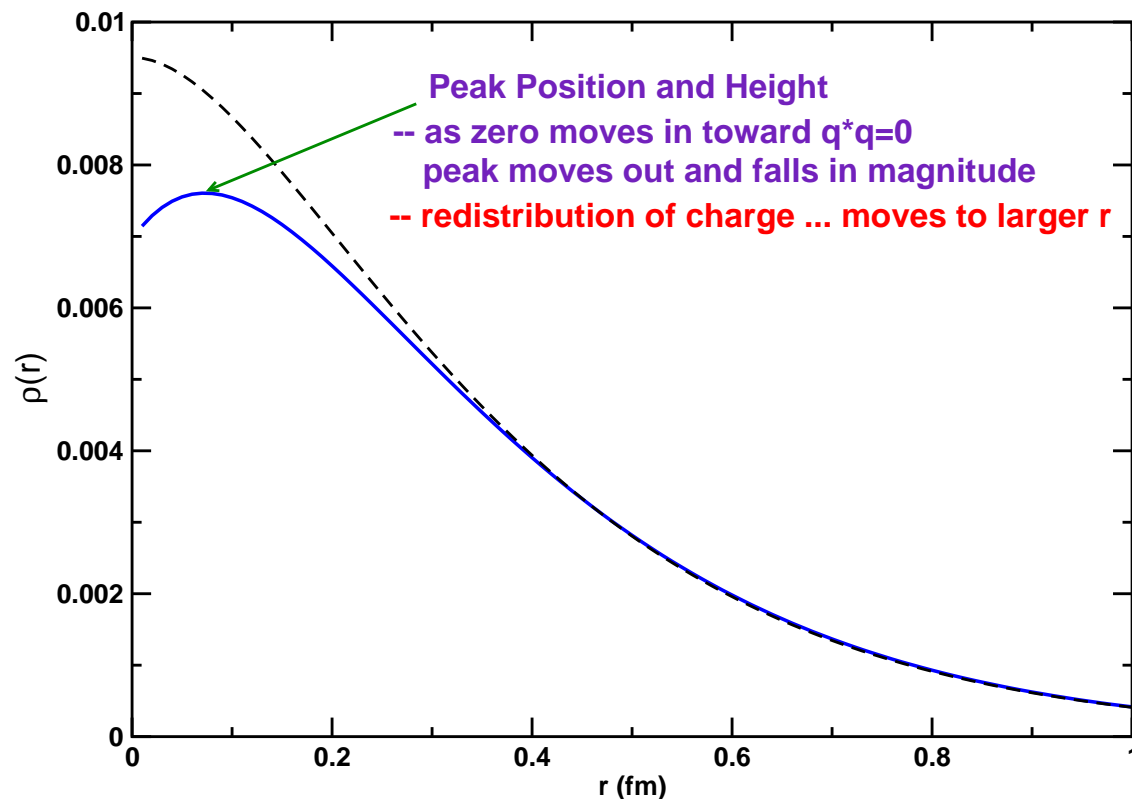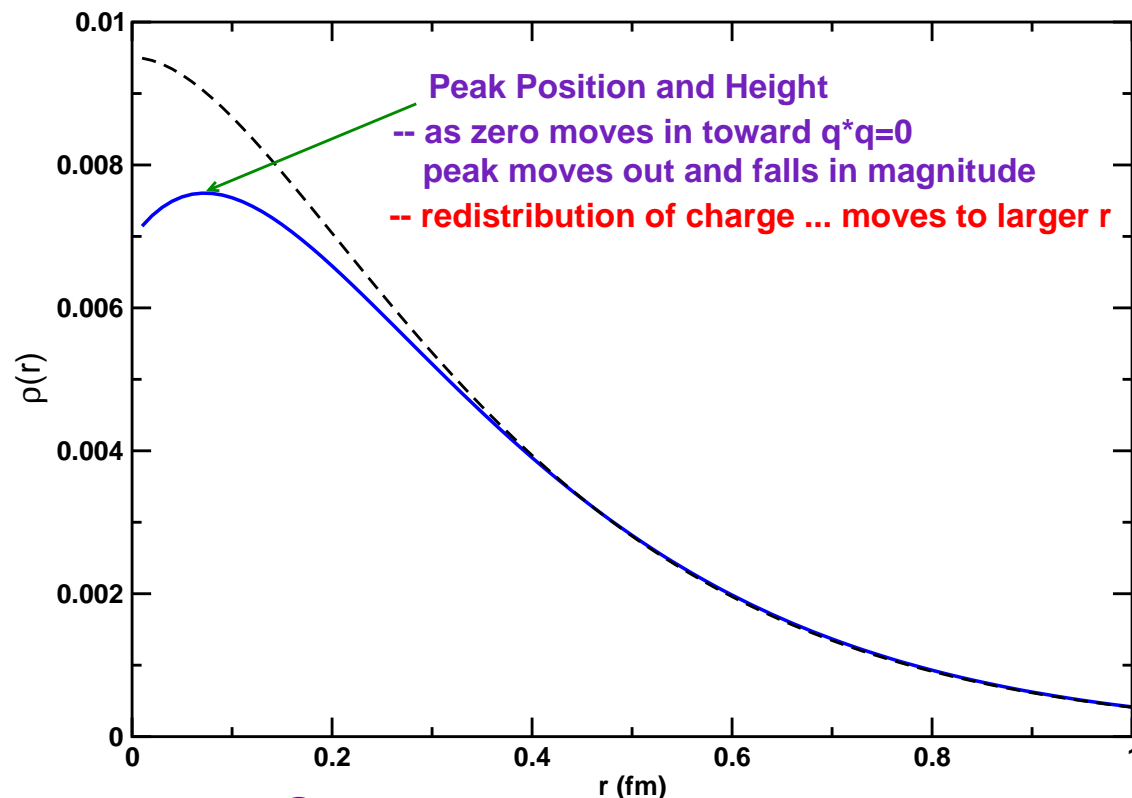
- However, Current Density remains peaked at $r = 0$!

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- Proton's Electromagnetic Form Factor

    - Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected

- Wave Function is complex and correlated mix of virtual particles and antiparticles: $s-$, $p-$ and $d-$waves

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- Proton's Electromagnetic Form Factor

  - Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected

- Wave Function is complex and correlated mix of virtual particles and antiparticles: $s-$, $p-$ and $d-$waves

- *Simple independent-particle three-quark bag-model picture is profoundly incorrect*

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Improved current

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Improved current

- 🔴 Composite axial-vector diquark correlation

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Improved current

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  - Improved performance of code

  - Implemented corrections so that large-Q^2 behaviour of form factors could be reliably calculated

  - Exposed two weaknesses in rudimentary *Ansatz*

    - Diquark effectively pointlike to hard probe

    - Didn't account for diquark being off-shell in recoil

Improved current

- Composite axial-vector diquark correlation

- Minor but material improvements to current

  - Introduce form factor: radius $0.8\,\mathrm{fm}$

  - Increase recoil mass by 10%

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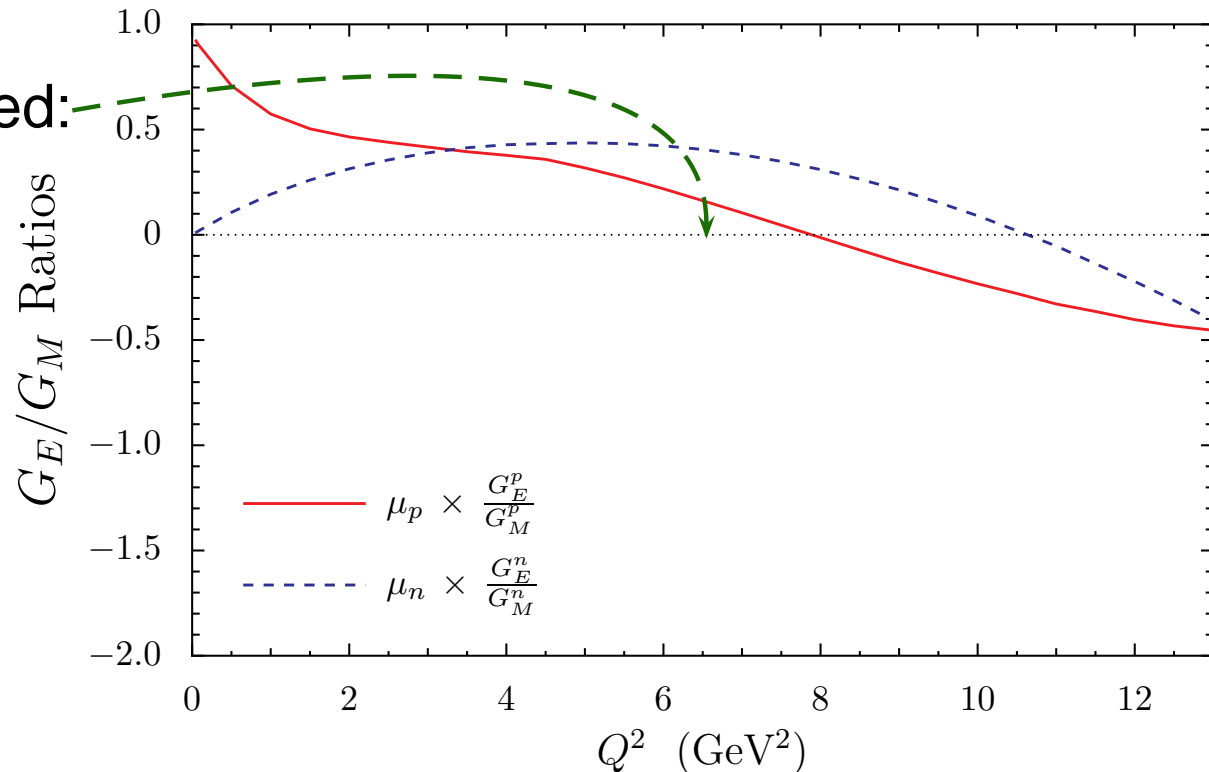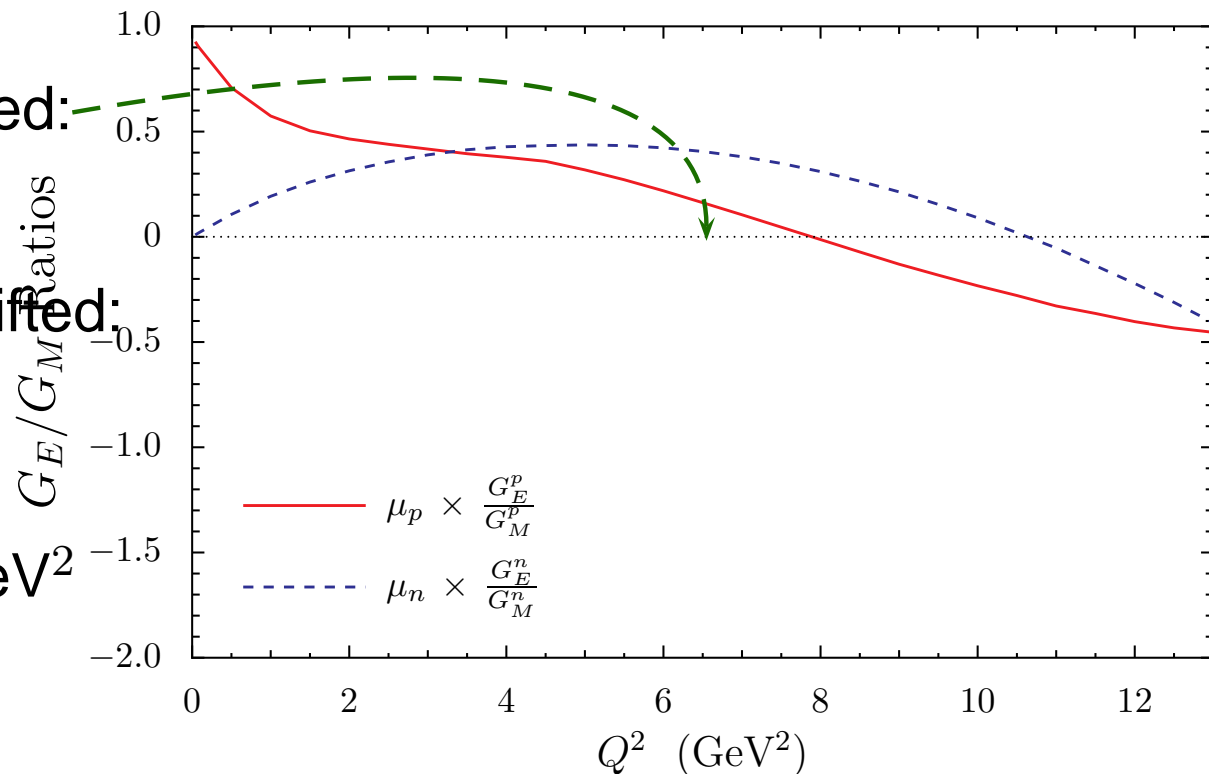
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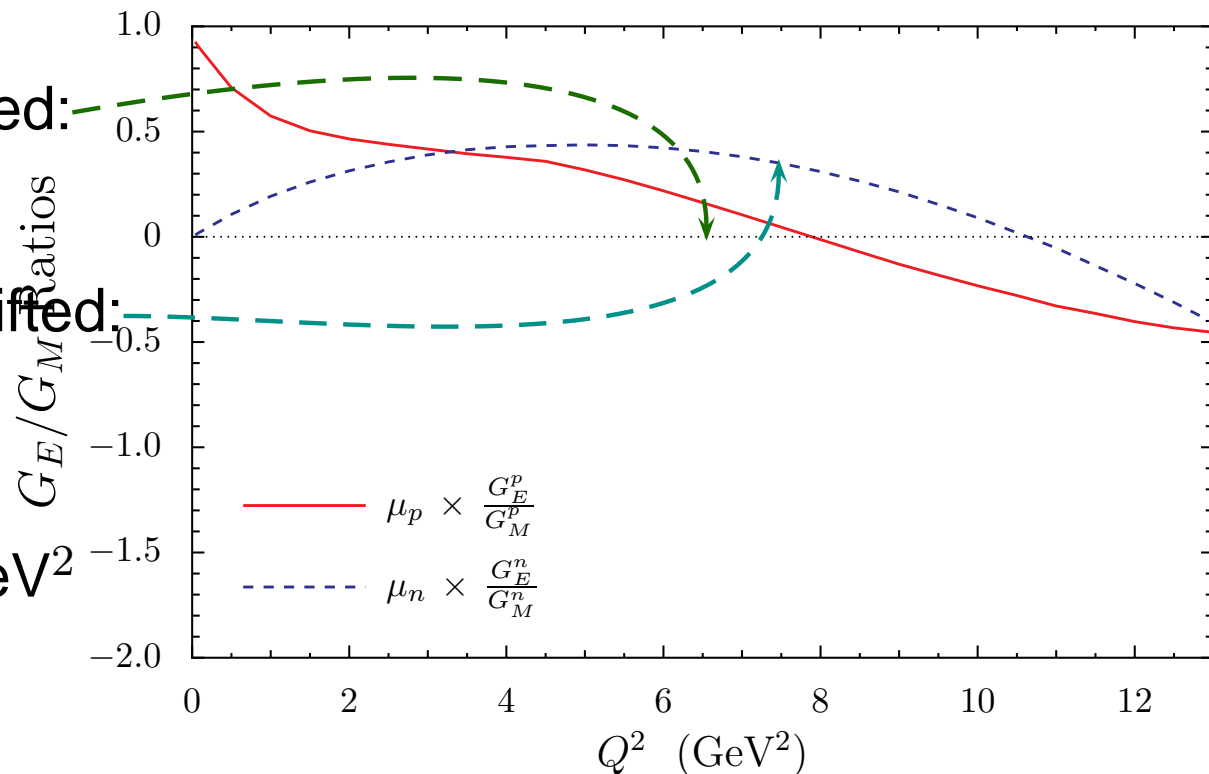
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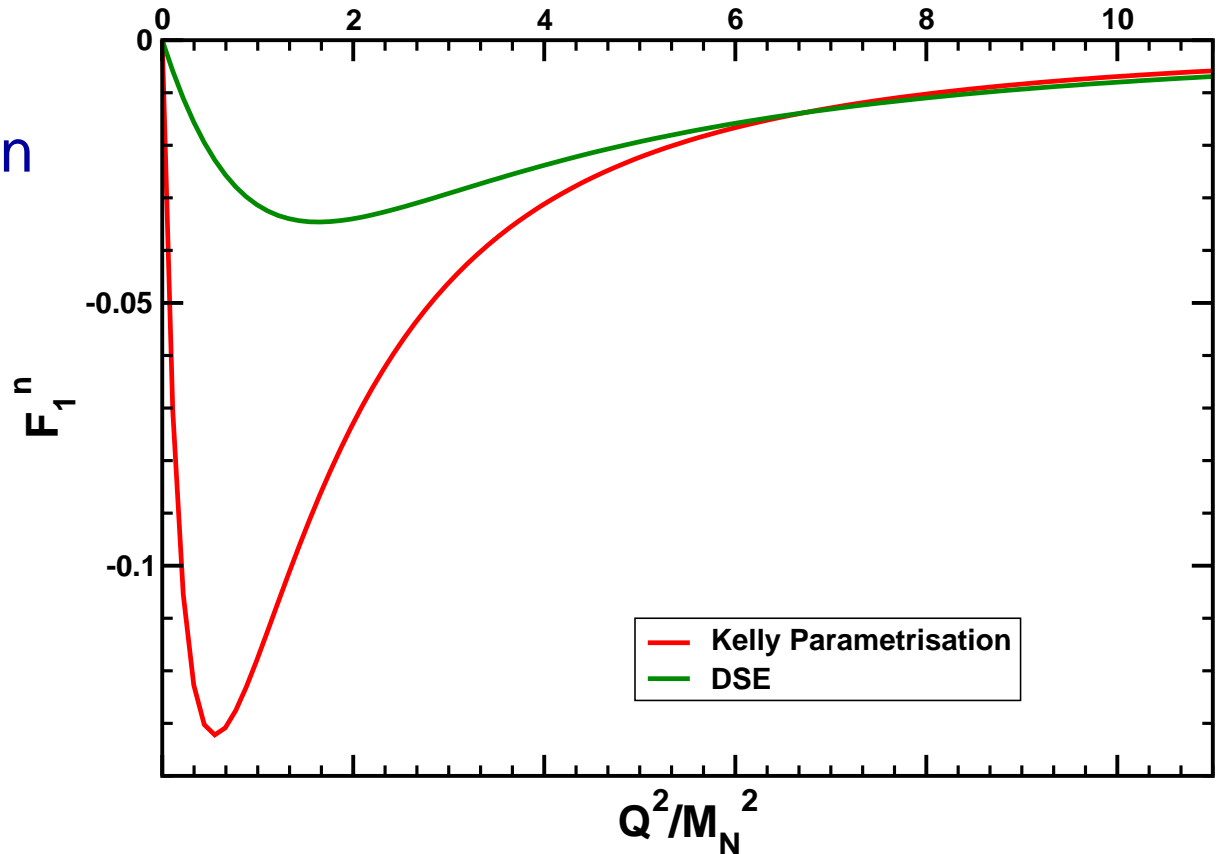 Neutron – peak shifted: $7.5 \rightarrow 5.0\,\mathrm{GeV}^2$ & now predict zero a little above $11\,\mathrm{GeV}^2$

The plot shows G_E/G_M Ratios versus Q^2 (GeV^2):
- $\mu_p \times \dfrac{G_E^p}{G_M^p}$ (red solid)
- $\mu_n \times \dfrac{G_E^n}{G_M^n}$ (blue dashed)

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Graph axes: vertical axis labelled G_E/G_M Ratios, horizontal axis labelled Q^2 (GeV^2).

Legend:
$\mu_p \times \dfrac{G_E^p}{G_M^p}$
$\mu_n \times \dfrac{G_E^n}{G_M^n}$

Pion Cloud

Comparison between Faddeev equation result and Kelly's parametrisation

$$Q^2/M_N^2$$

Kelly Parametrisation
DSE

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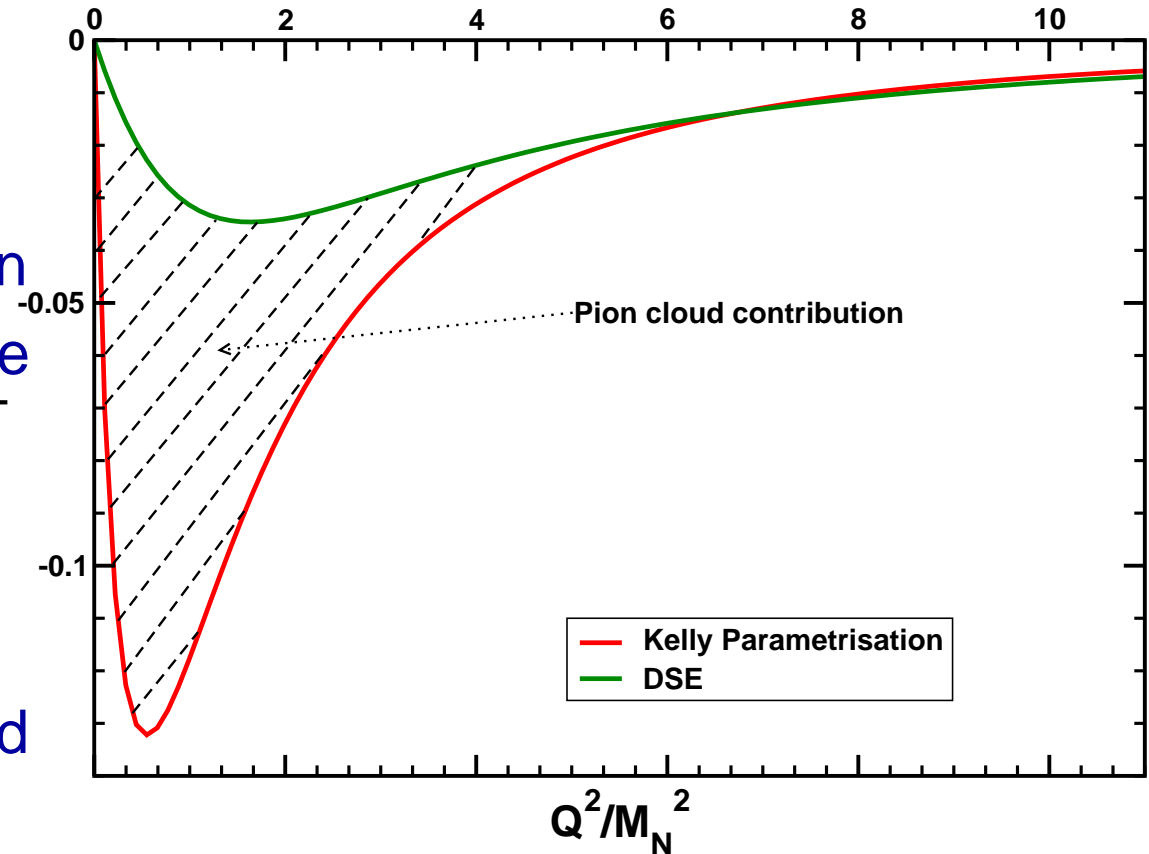25th Students' Workshop on Electromagnetic Interactions, 31/08 – 05/09, 2008. . . – p. 35/38

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Pion cloud contribution

Kelly Parametrisation
DSE

F_1^n

Q^2/M_N^2

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Epilogue

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DCSB exists in QCD.

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- DCSB exists in QCD.

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    - pseudoscalar mesons couple unnaturally strongly to light-quarks

    - pseudscalar mesons couple unnaturally strongly to the lightest baryons

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Epilogue

- **DCSB** exists in QCD.

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  - It impacts dramatically upon observables.

Epilogue

- Form Factors - progress anticipated in near- to medium-term
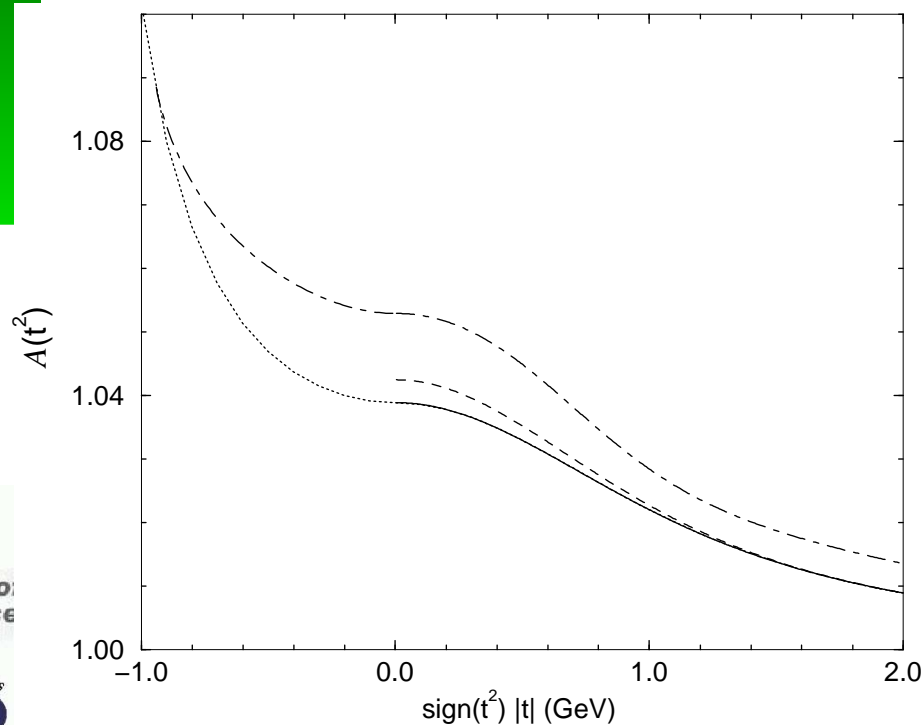 - Quantifying pseudoscalar meson "cloud" effects

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Vector piece of nucleon's self energy

$$M = 0.94,\ m_\pi = 0.14$$

$$\Lambda = 0.9\,\text{GeV},\ g_A = 1$$

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- Solid line: One-loop; numerically evaluated, exact (numerical) kernel

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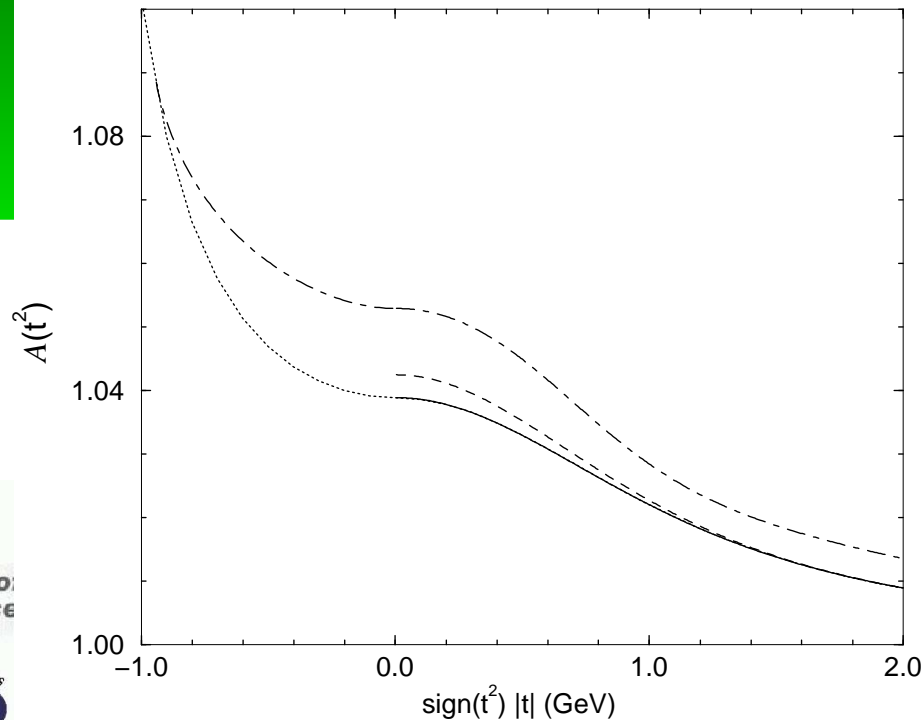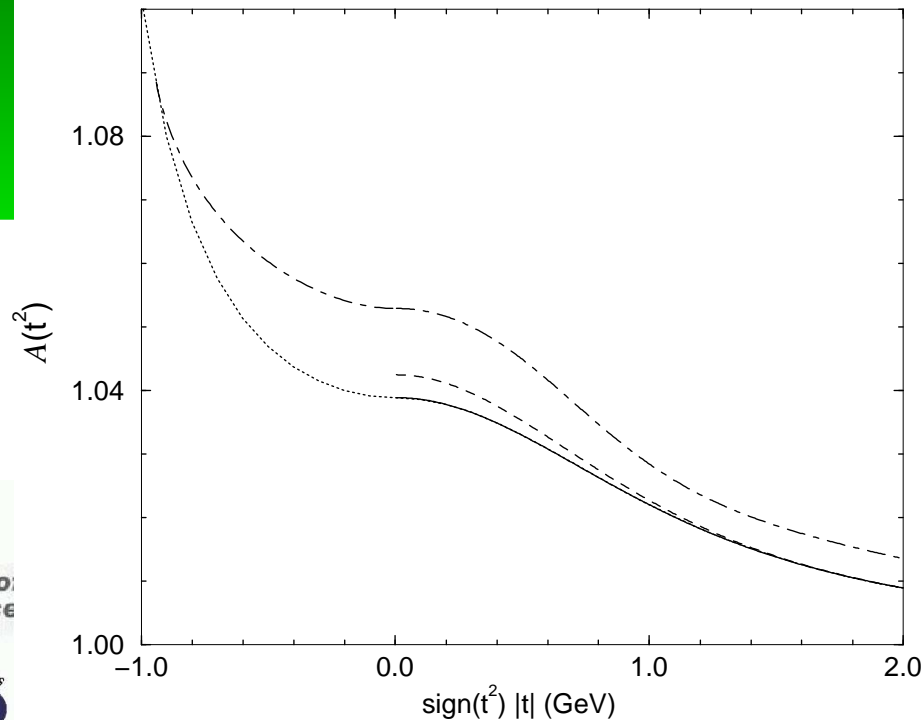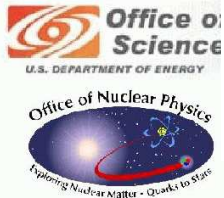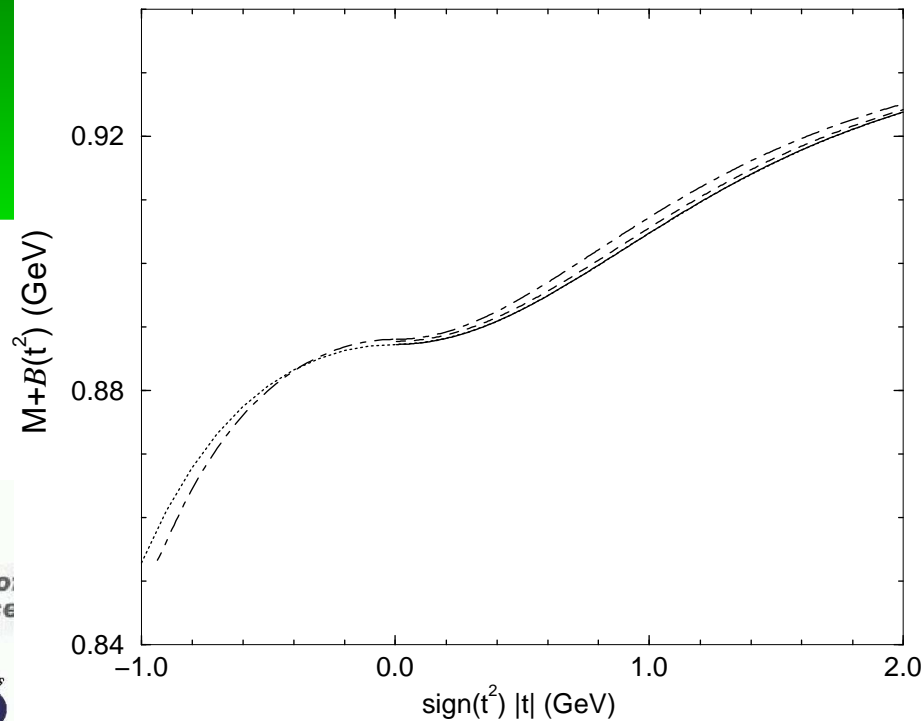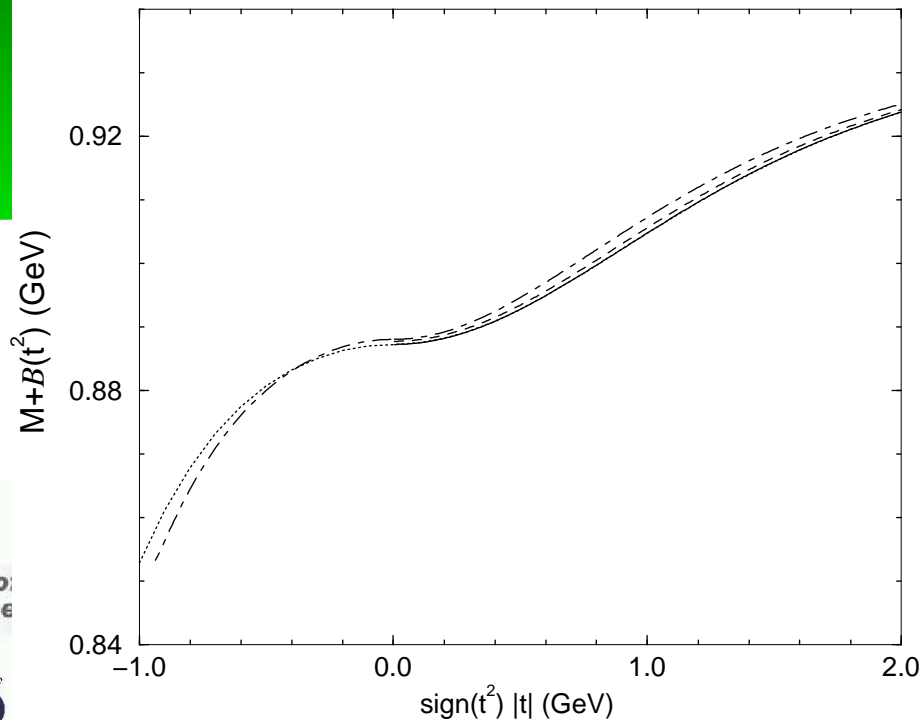
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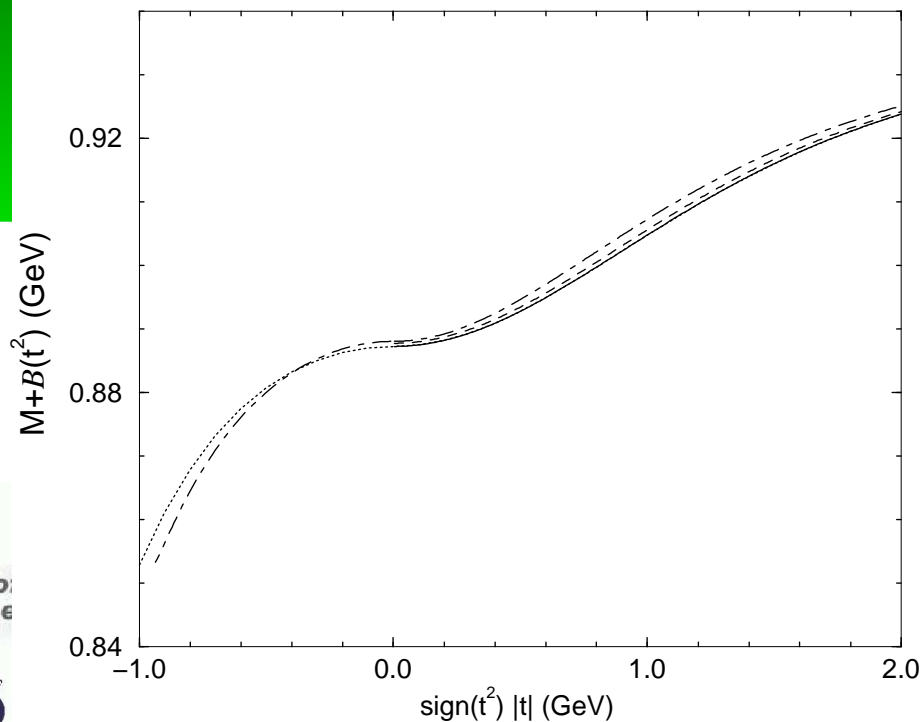
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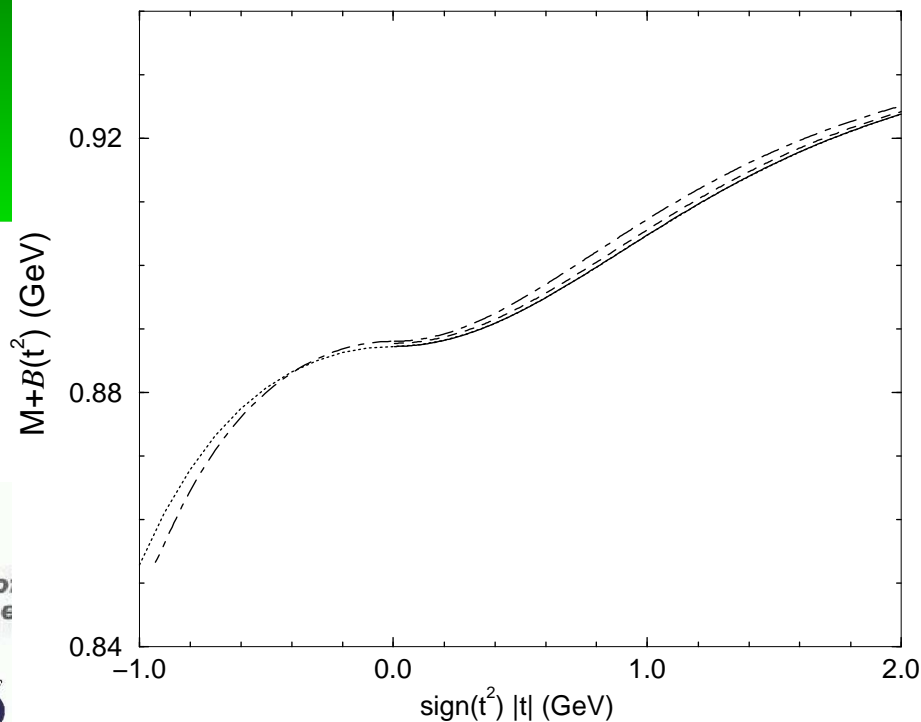
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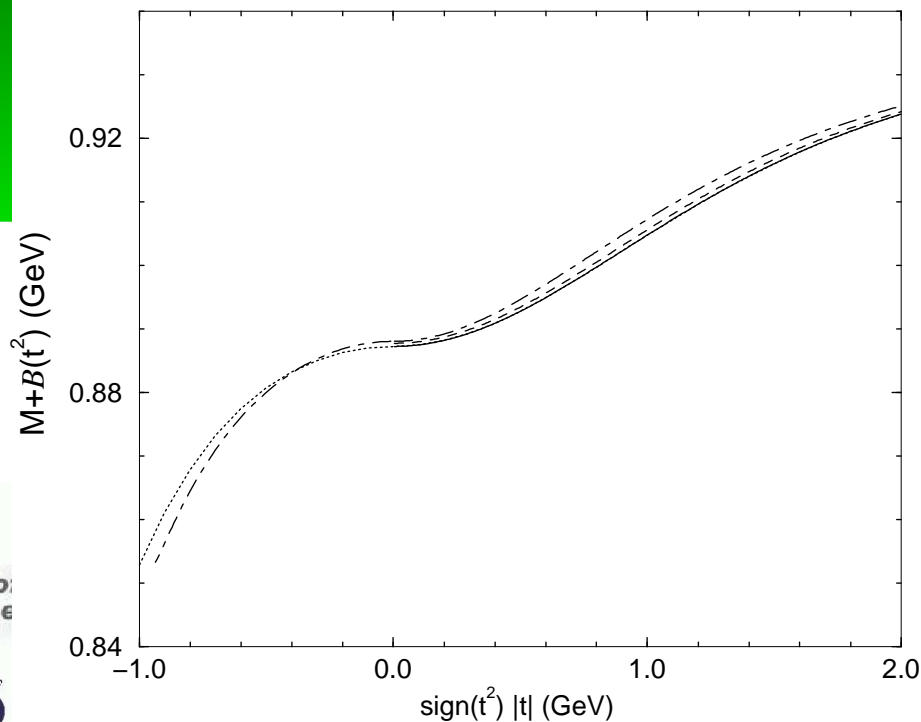
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